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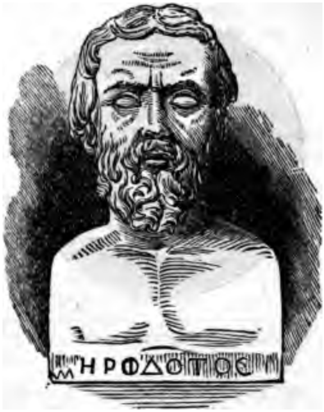

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A KEY

TO THE NEW AND PREVIOUS EDITIONS OF

A MANUAL OF MECHANICS.

BY

THE REV. JOSEPH A. GALBRAITH, A.M.,
FELLOW AND TUTOR OF TRINITY COLLEGE, DUBLIN,

AND

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The Reader is requested to omit the following examples in the Key (which are solutions to Problems in an earlier Edition), as unnecessary to be read.

Page 4	of Key.	Examples 1, 2.
" 5	"	" 3.
" 7	"	" 10.
" 8	"	" 13, 14, 16.
" 9	"	" 17, 19.
" 10	"	" 20, 21, 22.
" 11	"	" 2 at foot of page.
" 12	"	" 3.
" 14	"	" 4.
" 16	"	" 2, 3.
" 17	"	" 4, 5.
" 18	"	" 6.
" 20	"	" 11.

When the *word* "Key" does not follow the *word* "Fig.," it will be found in Plate II. at the end of the Key.

The solutions of the new questions which have been introduced into the Second and Third Editions of the Manual are inserted at the end of this New Edition of the Key.

INDEX.

N. B.—The numerals in the first two columns indicate the page and number of the Example in the fourth edition of the "Manual;" and those in the third column the page of the "Key" where the solution of the corresponding Example may be found. The fourth column gives the number of the Example in the "Key" *only* when it differs from the number of the corresponding Example in the "Manual."

A KEY TO THE MECHANICS.

EXPLANATION OF ABBREVIATIONS EMPLOYED IN THE FOLLOWING WORK.

$\angle$ signifies angle.

Δ „ triangle.

$\|^{m}$ „ parallelogram.

The pages and figures (unless "Key" follow) refer to the Rev. Messrs. Galbraith and Haughton's work; and a numeral in parentheses refers to the number of the Example.

STATICS.

Page 3.

(1.) We have only to reduce to lbs.

$$\begin{array}{rl}
1 \text{ ton, } 11 \text{ cwt. } 2 \text{ qrs. } 15 \text{ lbs.} & \\
20 & \\
\hline
31 & \text{cwt.} \\
4 & \\
\hline
126 & \text{qrs.} \\
28 & \\
\hline
1013 & \\
253 & \\
\hline
3543 & \text{lbs.} \quad \textit{Ans.}
\end{array}$$

(2.) We have only to divide 3543 lbs. by the number of lbs. in 1 ton, viz. by 2240, thus:

$$\begin{array}{l}
2240)\ 3543.\ (1.58169 \text{ tons. } \textit{Ans.} \\
\quad 2240 \\
\quad \overline{13030} \\
\quad 11200 \\
\quad \overline{\ \ 18300} \\
\quad \ \ 17920 \\
\quad \overline{\ \ \ \ 3800} \\
\quad \ \ \ \ 2240 \\
\quad \overline{\ \ \ \ 15600} \\
\quad \ \ \ \ 13440 \\
\quad \overline{\ \ \ \ \ 21600} \\
\quad \ \ \ \ \ 20160 \\
\quad \overline{\ \ \ \ \ \ \ 1440}
\end{array}$$

(3.) We must reduce both to the same denomination, viz. ounces, and *then* divide the former by the latter, thus:

```
25 lbs. 3 oz.          4 stones.
16                    14
---                   ---
153                   56 pounds.
25                    16
---                   ---
403 oz.               336
                      56
                      ---
                      896 ounces.
```

```
896) 403.0 (.44977   Ans.
     3584
     ----
      4460
      3584
      ----
       8760
       8064
       ----
        6960
        6272
        ----
         6880
         6272
         ----
          608
```

Page 6.

(1.) In fig. 9, suppose OA, OB the two equal forces, *making the* ∠ AOB = 120°. Now, since AG (= OB) = *AO, the* ∠ AGO = AOG, but AGO = BOG, as AG

is ∥ to OB, ∴ AOG = BOG = 60°; and since the 3 ∠s of △ AGO = 180°, it is equilateral, i. e. the resultant OG is = either component OA or OB.

Page 7.

(2.) Fig. 1 (Key). OB is the less component, OA the greater, and M the middle point of the resultant OG. Since OAG is a rt ∠, the semicircle on OG will pass through A, and ∴ AM = MG = GA, ∴ ∠ GOB, which is equal to AGM, = 60°, and ∴ AOG = 30°.

(4.) $\left.\begin{array}{r} 30^2 = 900 \\ 40^2 = 1600 \end{array}\right\}$ add, and *then* extract the square root of the sum.

```
25'00(50  Ans.
25
--
  00
```

(5.) $\left.\begin{array}{r} 43^2 = 1849 \\ 22^2 = 484 \end{array}\right\}$ add, and extract the square root.

```
       23'33(48.3011 lbs.  Ans.
       16
       --
    88| 733
     8| 704
   ---|----
   963| 2900
     3| 2889
 -----|-----
 96601| 110000
     1|  96601
------|-------
966021| 1339900
      |  966021
      |  -------
```

(6.) $\left.\begin{array}{l} 56^2 = 3136 \\ 31^2 = 961 \end{array}\right\}$ subtract and extract the square root of the difference.

$$\begin{array}{r|l}
 & 21'75(46.6368 \text{ lbs. } Ans. \\
 & 16 \\
\hline
86 & 575 \\
6 & 516 \\
\hline
926 & 5900 \\
6 & 5556 \\
\hline
9323 & 34400 \\
3 & 27969 \\
\hline
93266 & 643100 \\
6 & 559596 \\
\hline
932728 & 8350400 \\
 & 7561824 \\
\hline
\end{array}$$

(9.)

$$\begin{array}{r}
\cos 12^\circ = .97815 \\
142 \\
\hline
195630 \\
391260 \\
97815 \\
\hline
138.89730
\end{array}$$

138.89730 lbs. *Ans.*

(10.) 1 cwt. 2 qrs. 14 lbs. = 182 lbs.

$182 \times .97815 = 178.0233$ lbs.
= 1 cwt. 2 qrs. 10.0233 lbs. *Ans.*

(11.) $\cos 17^\circ = .9563$. Dividing 120 lbs. by this, we *obtain* 125.483 lbs. *Ans.*

(12.)

$$\cos 37^\circ = .79863$$
$$\cos 53^\circ = .60181$$

Multiplying these cosines by 471, we obtain 376.15473 and 283.45251.

(13.) 3 cwt. 2 qrs. 17 lbs. = 409 lbs.

$$\left.\begin{array}{l}\cos 17^\circ = .9563 \\ \cos 73^\circ = .29237\end{array}\right\} \text{Multiplying these by 409, we obtain respectively—}$$

391.1267 lbs. and 119.57933. *Ans.*

(14.) We have only to reduce the answer of the last example to cwts.

Page 8.

(16.)

$$17^2 = 289$$
$$36^2 = 1296$$
$$\left.\begin{array}{r}\cos 22^\circ = .92718 \\ 2 \times 17 \times 36 = 1224\end{array}\right\} \text{product of these} = 1134.86832$$

$$\left.\begin{array}{r}289 \\ 1296 \\ 1134.86832\end{array}\right\} \text{add and extract the square root of the sum.}$$

$$\begin{array}{r|l}
 & 27'19.86'83'20(52.152 \quad \textit{Ans.} \\
 & 25 \\ \hline
102 & 219 \\
2 & 204 \\ \hline
1041 & 1586 \\
1 & 1041 \\ \hline
10425 & 54583 \\
5 & 52125 \\ \hline
104302 & 245820 \\
 & 208604 \\ \hline
\end{array}$$

17.)

$$26^2 = 676$$
$$127^2 = 16129$$
$$\cos 76^\circ = .24192$$

$$\left.\begin{array}{r} 2 \times 26 \times 127 \times .24192 = 1597.63968 \\ 676. \\ 16129. \end{array}\right\} \text{add}$$
$$\overline{18402.63968}$$

The square root of this is 135.656 *Ans.*

(19.) Fig. 5. OA and OB the components, and OC the resultant.

$$74^2 = 5476$$
$$123^2 = 15129$$
$$\cos 65^\circ = .42262$$

$$2 \times 74 \times 123 \times .42262 = 7693.37444$$

$$\left.\begin{array}{r} 7693.37444 \\ 15129. \\ 5476. \end{array}\right\} \text{add}$$
$$\overline{28298.37444}$$

The square root of this is $168.221 = OC$.

Now in Δ AOC we have

$$OC : CA :: \sin CAO : \sin AOC$$

but $\sin CAO = \sin AOB$, since they are supplemental $\angle^s$.

$$\therefore 168.221 : 74 :: \sin 65^\circ : \sin AOC$$
$$\text{or } 168.221 : 74 :: .90631 : \sin AOC$$

multiplying the second and third terms of this proportion together, and dividing by the first, we obtain

$$\sin AOC = .398689 = \sin 23^\circ\ 30'$$

and subtracting this $\angle$ from 65° gives $41^\circ\ 30'$.

(20.) Fig. 5. Here $\cos 132^\circ = -\cos$, its supplement, $= -\cos 48^\circ = -.66913$.

$$11 \text{ cwt. } 2 \text{ qrs. } 13 \text{ lbs.} = 1301 \text{ lbs.} = OA$$
$$5 \;\; " \;\; 3 \;\; " \;\; 17 \;\; " = 661 \;\; " = OB$$

$$\left.\begin{array}{r} 1301^2 = 1692601 \\ 661^2 = \underline{436921} \end{array}\right\} \text{add}$$
$$2129522$$

$$2 \times 1301 \times 661 \times .66913 = 1150851.40786$$

This must be subtracted from the above sum since the cosine is negative.

$$\left.\begin{array}{r} 2129522.00000 \\ \underline{1150851.40786} \end{array}\right\} \text{subtract}$$
$$978670.59214$$

The square root of this, viz. $989.277 = OC$, the resultant.

Now in Δ AOC we have

$$OC : CA :: \sin OAC : \sin COA$$
$$\text{or } 989.277 : 661 :: \sin 48^\circ : \sin COA$$
$$\text{or } 989.277 : 661 :: .74314 : \sin COA$$

Multiplying the second and third terms together, and dividing by the first, we obtain,

$$\sin COA = .49653 = \sin 29^\circ\ 46'$$

which $\angle$ being taken from 132° leaves $102^\circ\ 14'$, the other required angle.

(21.) The component forces are plainly (see fig. 5, where in Δ AOC, OA : AC or OB : : sin ACO or sin BOC : sin AOC) in the same ratio as $\sin 6^\circ : \sin 31^\circ$; hence we have only to divide $\sin 6^\circ = .10453$ by $\sin 31^\circ = .51504$ The result true to six places is .202955

(22.) Fig. 5. Here $OA = 22$, $OC = 56$, and the $\angle$ $AOC = 15^\circ$; we have $\therefore$ only to find OB or AC.

Now $AC^2 = AO^2 + CO^2 - 2 \times AO \times CO \times \cos AOC$.

$$\left.\begin{array}{r} 22^2 = 484 \\ 56^2 = \underline{3136} \end{array}\right\} \text{add}$$
$$3620$$

$$\cos 15^\circ = .96592$$
$$2 \times 22 \times 56 \times .96592 = 2380.02688$$

$$\left.\begin{array}{r} 3620.00000 \\ 2380.02688 \end{array}\right\} \text{subtract}$$
$$\overline{1239.97312}$$

The square root of this, viz. 35.213, is the required component.

N. B.—This question should more properly have been classed under 15.

Page 10.

(1.)
$$\begin{array}{r} 220 \\ 126 \\ \hline 346 : 220 :: 26 \text{ inches.} \end{array}$$

This proportion gives 16.53 inches, the length of the segment *remote* from the weight 220 lbs.

(2.)
$$\begin{array}{rr} 17 \text{ st. } 6 \text{ lbs.} = & 244 \text{ lbs.} \\ 2 \text{ cwt. } 1 \text{ qr. } 22 \text{ lbs.} = & 274 \text{ lbs.} \\ \hline & 518 : 244 :: 32 \text{ feet.} \end{array}$$

This proportion gives 15.073, the length of the segment *next* the weight 2 cwt. 1 qr. 22 lbs.

Page 13.

(2.) Fig. 16.
$$OA = 31$$
$$OB = 16$$
$$OC = 29$$
$$\cos 17^\circ = .9563$$

$$\left.\begin{array}{r} 31^2 = 961. \\ 16^2 = 256. \\ 2 \times 16 \times 31 \times .9563 = 948.6496 \end{array}\right\} \text{add}$$
$$\overline{2165.6496}$$

The square root of this is $46.536 = OG$.

Now, $OG : GB :: \sin GBO : \sin GOB$; but $\sin GBO = \sin AOB$, since these $\angle^s$ are supplemental; hence the proportion becomes $46.536 : 31 :: \sin 17° : \sin GOB$; or, $46.536 : 31 :: .29237 : \sin GOB$ $\therefore \sin GOB = .19476 = \sin 11° 14'$.

$$\begin{array}{r} 11° \ 14' \\ 52 \quad 0 \\ \hline \end{array}$$
$$\therefore 63° \ 14' = GOC,$$
$$\text{and } \cos 63° \ 14' = .45036$$
$$\sin 63° \ 14' = .89285$$

Again,
$$\begin{array}{r} OG^2 = 2165.6496 \text{ (found before)} \\ OC^2 = 29^2 = 841. \\ 2 \times 29 \times 46536 \times .45036 = 1215.56127 \\ \hline 4222.21087 \end{array}$$

The square root of this is *very nearly* $64.98 = OH$.

Again, from Δ HOC, we have

$OH : HC \ (= OG) :: \sin HCO \ (= \sin GOC) : \sin HOC$.
That is, $64.98 : 46.536 :: \sin 63° \ 14' : \sin HOC$.
Or, $64.98 : 46.536 :: .89285 : \sin HOC$.
$\therefore \sin HOC = .63942 = \sin 39° \ 45'$.

(3.) Fig. 16.
$$\begin{array}{r} OA = 217 \text{ lbs.} \\ OB = 300 \\ OC = 167 \\ \cos 17° = .9563 \end{array}$$

$$\left.\begin{aligned} 217^2 &= 47089. \\ 300^2 &= 90000. \\ 2 \times 217 \times 300 \times .9563 &= 124510.26 \end{aligned}\right\}\text{add}$$

$$\overline{261599.26}$$

The square root of this is $511.467 = OG$,

Now, $OG : GB :: (\sin OBG =) \sin AOB : \sin GOB$.
That is, $511.467 : 217 :: \sin 17^\circ : \sin GOB$.
Or, $511.467 : 217 :: .29237 : \sin GOB$.
$\therefore \sin GOB = .12404 = \sin 7^\circ\ 8'$.

$$\left.\begin{matrix} 7^\circ\ 8' \\ 15^\circ \end{matrix}\right\}\text{add}$$

$$\overline{22^\circ\ 8' = GOC.}$$
$$\cos 22^\circ\ 8' = .92631$$
$$\sin 22^\circ\ 8' = .37676$$

Again,

$$\left.\begin{aligned} 2 \times 167 \times 511.467 \times .92631 &= 158241.51692118 \\ 167^2 &= 27889. \\ OG^2 &= 261599.26 \end{aligned}\right\}\text{add}$$

$$OH^2 = 447729.77692118$$

$$\therefore OH = 669.126 \text{ lbs.}$$

Again,

$HO : OC :: (\sin HCO =) \sin GOC : (\sin CHO =) \sin HOG$.
That is, $669.126 : 167 :: \sin 22^\circ\ 8' : \sin HOG$.
Or, $669.126 : 167 :: .37676 : \sin HOG$.
$\therefore \sin HOG = .09403 = \sin 5^\circ\ 24'$.

$$\left.\begin{aligned} GOC &= 22^\circ\ 8' \\ HOG &= 5^\circ\ 24' \end{aligned}\right\}\text{subtract}$$

$$\therefore HOC = 16^\circ\ 44'$$

N. B.—Since $BOC = 15^\circ$, and $HOC = 16^\circ\ 44'$, it is *clear that* BO ought to be on the side of OH next OC

*

(4.) Fig. 16.

$$OA = 22 \text{ lbs.}$$
$$OB = 16$$
$$OC = 100$$
$$AOB = 37^\circ$$
$$BOC = 69^\circ$$
$$\cos 37^\circ = .7986355$$
$$\sin 37^\circ = .6018150$$

$$\left.\begin{aligned} 2 \times 22 \times 16 \times .7986355 &= 562.239392 \\ 22^2 &= 484. \\ 16^2 &= 256. \end{aligned}\right\} \text{add}$$

$$OG^2 = 1302.239392$$

$$\therefore OG = 36.0865$$

Again,

OG : GB : : (sin GBO =) sin AOB : sin BOG.

That is, 36.0865 : 22 : : .601815 : sin BOG.

$\therefore$ sin BOG = .3668942 = sin $21^\circ\ 31'\ 27''$.

$$\left.\begin{aligned} BOG &= 21^\circ\ 31'\ 27'' \\ BOC &= 69^\circ \end{aligned}\right\} \text{add}$$

$\therefore$ GOC $= 90^\circ\ 31'\ 27''$, and $\therefore$ HGO $= 89^\circ\ 28'$.

Now, $\sin 90^\circ\ 31'\ 27'' = \sin 89^\circ\ 28'\ 33''$ (its supplement)

$$= .9999581$$

$$\cos 90^\circ\ 31'\ 27'' = -\cos 89^\circ\ 28'\ 33'' = -.0091483$$

Again,

$$\left.\begin{aligned} OG^2 &= 1302.239392 \\ 100^2 &= 10000. \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &11302.239392 \\ 2 \times 100 \times 36.0865 \times .0091483 &= 66.02602559 \end{aligned}\right\} \text{subtract}$$

$$OH^2 = 11236.21236641$$
$$\therefore OH = 106.001$$

Again, *OH : HG* : : sin HGO : sin HOG.

That is, 106.001 : 100 : : .9999581 : sin HOG.

$\therefore \sin HOG = .9433478 = \sin 70^\circ\ 37'\ 18''$.

Now, $\left.\begin{matrix} GOC = 90^\circ\ 31'\ 27'' \\ HOG = 70^\circ\ 37'\ 18'' \end{matrix}\right\}$ subtract

$\therefore HOC = 19^\circ\ 54'\ 9''$

N. B.—I may here remark, *once for all*, that I have used Chambers' Tables in the work of the above question, and occasionally elsewhere, when greater accuracy was aimed at than could well be secured by the too limited Tables at the end of the *Manual.*

Page 15.

(2.) Fig. 18. Since CD : AB : : PO : OQ
$\therefore$ CD + AB : AB : : PO + OQ : OQ
That is, 65 lbs. : 23 lbs. : : 14 in. : OQ.
$\therefore$ OQ = 4.9538 in. *Ans.*

(3.) Fig. 18.
3 cwt. 2 qrs. 15 lbs. = 407 pounds.
1 „ 3 „ 25 „ = 221

628 : 221 : : 3 ft. 7 in.
12

43

$\therefore$ OP = 15.132 in. = 1.261 feet.

Page 16.

(5.) "It is not difficult to prove that the centre of gravity is the centre of the circle inscribed in the triangle OPQ." (Fig. 2, Key.)
Since AQ = QB and BO = OC, &c.

$\therefore$ AQ : QB : : CO : OB, and $\therefore$ (Euclid, B. vi. p. 2.) QO is parallel to AC, i. e. the line joining middle points of two sides of a Δ is parallel to the third side, $\therefore$ AO, BP, CQ are $\|^{ms}$, and $\therefore$ QO = AP = PC = $\frac{1}{2}$ AC, &c. Now, cut QO in X so that QX : XO : : BC : BA, i. e.

$$QX : XO : : PQ \left(= \tfrac{1}{2} BC\right) : PO \left(= \tfrac{1}{2} BA\right)$$

$\therefore$ PX bisects the $\angle$ OPQ (Euclid, B. vi. p. 3), but centre of gravity lies in PX. Similarly it can be shown to lie in OZ, which bisects the $\angle$ POQ; $\therefore$ G is the centre of gravity, but the bisectors of the $\angle^s$ of a Δ meet in centre of inscribed circle. (Euclid, B. iv. p. 4).

Page 20.

(2.) 13 in. : 2 in. : : 2 cwt.

$$\begin{array}{r} 112 \\ \hline 224 \\ 2 \\ \hline 13)448(34.4615 \text{ lbs. } \textit{Ans.} \end{array}$$

(3.) $\cos 126^\circ = -\cos 54^\circ$ (its supplement)

$$\begin{aligned} &= -.58778 \\ 217^2 &= 47089 \\ 725^2 &= 525625 \end{aligned} \Bigg\} \text{add}$$

$$\begin{array}{r} 572714 \\ 2 \times 725 \times 217 \times .58778 = 184944.977 \\ \hline 387769.023 \end{array} \Bigg\} \text{subtract}$$

The square root of this is 622.711 lbs., the required *strain on the fulcrum.*

Page 21.

(4.) Fig. 3 (Key).

Let OA (= 310 lbs.) represent the power, OB (= 422 lbs.) the resistance, then, the parallelogram AOBC being completed, its diagonal OC will represent the upward strain on the fulcrum.

Now, cos AOB = – cos POR = – cos 37°, since AOB + POR = 180°.

$$\cos 37^\circ = .79863$$

$$\left.\begin{aligned} 422^2 &= 178084 \\ 310^2 &= 96100 \end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned} &274184 \\ 2 \times 310 \times 422 \times .79863 = {}&208953.5532 \end{aligned}\right\}\text{subtract, since cos AOB is } \textit{negative.}$$

$$OC^2 = 65230.4468$$

$$\therefore OC = 255.402 \text{ lbs.} \quad \textit{Ans.}$$

(5.) Fig. 4 (Key).

Let OA (= 217 lbs.) represent the power, OB (= 100 lbs.) the resistance, then the diagonal OC will represent the *downward* strain on the fulcrum.

Now, cos AOB = – cos POR = – cos 42° = – .74314

$$\left.\begin{aligned} 217^2 &= 47089 \\ 100^2 &= 10000 \end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned} &57089. \\ 2 \times 100 \times 217 \times .74314 = {}&32252.276 \end{aligned}\right\}\text{subtract}$$

$$OC^2 = 24836.724$$

$$\therefore OC = 157.596 \text{ lbs.} \quad \textit{Ans.}$$

(6.) $\cos 79^\circ = .19081$

$$\left.\begin{array}{r} 2 \times 17 \times 32 \times .19081 = 207.60128 \\ 32^2 = 1024. \\ 17^2 = 289. \end{array}\right\} \text{add}$$

$$1520.60128$$

The square root of this is 38.9948 lbs., the strain on the fulcrum.

Page 23.

(1.) 2 ft. 8 in. : 6 in. : : 1156 lbs.

This statement gives for the fourth proportional 216.75 lbs. *Ans.*

(2.) 79 lbs. : 17 lbs. : : 17 in.

Hence the fourth term is 3.65822 in. *Ans.*

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(3.) 29 lbs. $\times$ 16 = 464 lbs.
1 ft. 2 in. : 8 ft. : : 464 lbs.

The fourth term is $3181\frac{5}{7}$ lbs. = 1 ton 8 cwt. 1 qr. $17\frac{5}{7}$ lbs. *Ans.*

(4.) 1 ft. 2 in. = 14 in. radius of axle, $\therefore$ 16 = radius of axle *after* the rope is wound round it. Now, the resistances capable of being overcome, the length of the lever being the same, are inversely as the radii of the axles, or as 16 : 14, i. e. if 16 represent the power of the capstan originally, 14 will *now* represent its *diminished* power, and the difference 2 is $\frac{1}{8}$ of 16 and 100 $\div$ 8 = $12\frac{1}{2}$.

(5.) 1 ton = 2240 lbs.
1 ft. 7 in. : 2.5 in. : : 2240 lbs.

The fourth term is 294.7368 lbs. *Ans.*

Page 26.

(1.) Denoting the inclination by i, we have

$$\sin i = \frac{1}{62} = .0161290 = \sin 55' \, 26''.6$$

(2.) $\sin 4^\circ = .06975$, l = length,
h = height.

$$\therefore l : h :: 1 : .06975$$

$$\text{or } l : h :: \frac{1}{.06975} : 1$$

Hence, $\frac{1}{.06975} = 14.3369$ is the required number.

Page 27.

(3.) 6 miles = 31680 feet.

$$\therefore \sin i = \frac{511}{31680} = .0161300 = \sin 55' \, 27''.1939.$$

Also, $\frac{511}{31680} = \frac{1}{61.996}$ which shows that the fall is 1 in 62 nearly.

(4.) A force equal to $\frac{1}{21}$ part of the weight overcomes friction, and to $\frac{1}{43}$, the resistance arising from the slope of the hill, abstracting from friction, $\therefore$ to overcome *both together* the force must be $\frac{1}{21} + \frac{1}{43} = \frac{64}{903} = \frac{1}{14.109}$ *Ans.*

(5.) $\frac{1}{200} + \frac{1}{56} = \frac{128}{5600} = \frac{1}{43.75}$ *Ans.*

(6.) 14 ft. : 3 ft. : : 964 lbs.

The fourth term is $206\frac{4}{7}$ lbs. *Ans.*

(7.) By the rule given at foot of page 20 of Manual, we have P : W : : sin 6° : sin 101°, or,

$$\sin 101^\circ : \sin 6^\circ :: W : P.$$

Now, sin 6° = .10453, and sin 101° = sin 79° (its supplement), = .98162.

2 cwt. 1 qr. 17 lbs. = 269 lbs.

$\therefore$ .98162 : .10453 : : 269 lbs. : P,

$\therefore$ P = 28.645 lbs. *Ans.*

(8.) Sin 120° = sin 60° = .86602 sin 2° = .03490

Now, sin 120° : sin 2° : : 31 ton : P,

That is, .86602 : .03490 : : 31 ton : P,

$\therefore$ P = 1.249278 ton = 1 ton 4 cwt. 3 qrs. 26.38272 lbs.

(9.) Sin 167° = sin 13° = .22495,

$\therefore$ .22495 : $\frac{1}{26}$: : 22 tons : P.

.22495 × 26 = 5.8487; dividing 22 by this we obtain,

P = 3.7615 tons. *Ans.*

(10.) Fig. 34. In Δ OR′V′, the $\angle$ OV′R′ is right, OR′ represents the weight of the waggon (= 2 ton 14 cwt. = 54 cwt), and OV′ the pressure upon the road, also R′ OV′ = i, whose sine = $\frac{1}{27}$ = .0370370 $\therefore$ $i = 2^\circ\ 7'\ 21''$, and cos i = .9993214 Hence, OV′ = OR′ cos i = 54 cwt. × .9993214 = 53.9633556 cwt. = 2 ton 13 cwt. 3 qrs. 23.89 lbs.

(11.) Fig. 36. In Ex. 7, we have,

$$\left.\begin{array}{l} OV'P' = 6^\circ \\ P'OV' = 101^\circ \end{array}\right\} \text{add}$$

$$\overline{107^\circ}$$

Taking this from 180°, we get OP′V′ = 73°; also, P′V′ = 2 cwt. 1 qr. 17 lbs. = 269 lbs. Hence, from Δ OP′V′ we obtain,

Sin P′OV′ : sin OP′V′ : : P′V′ : OV′,

That is, (sin 101° =) sin 79° : sin 73° : 269 lbs. : OV′,

Or, .98162 : .95630 : : 269 : OV′,

$\therefore$ OV′ = 262.06 lbs. = 2 cwt. 1 qr. 10.06 lbs.

In Ex. 8.
$$\left.\begin{array}{l} OV'P' = 2^\circ \\ P'OV' = 120^\circ \end{array}\right\} \text{add}$$

$$\overline{122^\circ}$$

Taking this from 180°, there remains the $\angle$ OP′V′ = 58°; also P′V′ = 31 tons.

$$\therefore \sin P'OV' : \sin OP'V' :: P'V' : OV',$$
That is $(\sin 120^\circ =) \sin 60^\circ : \sin 58^\circ :: 31 \text{ tons} : OV'$,
$$\text{Or, } .86602 : .84805 :: 31 : OV',$$
$$\therefore OV' = 30.3567 \text{ tons.}$$

In Ex. 9. $\sin OV'P' = \frac{1}{26} = .0384615 = \sin 2^\circ\ 12'\ 15''$.

$$\left.\begin{array}{l} P'OV' = 167^\circ \\ OV'P' = \quad 2^\circ\ 12'\ 15'' \end{array}\right\}\text{add}$$
$$\overline{169^\circ\ 12'\ 15''}$$

The difference between this and 180°, viz. $10^\circ\ 47'\ 45''$ = OP′V′; also P′V′ = 22 tons.

$$\therefore \sin P'OV' : \sin OP'V' :: P'V' : OV',$$

That is,

$(\sin 167^\circ =) \sin 13^\circ : \sin 10^\circ\ 47'\ 45'' :: 22 \text{ tons} : OV'$,
$$\text{Or, } .2249511 : .1873099 :: 22 : OV',$$
$$\therefore OV' = 18.318727 \text{ tons}$$
$$= 18 \text{ tons } 6 \text{ cwt } 1 \text{ qr. } 13.94848 \text{ lbs.}$$

The result in the Manual is incorrect.

(12.) 1 ton = 2240 lbs.

To equilibrate a weight of 2240 lbs. on a road of gradient, 1 in 21 will require, abstracting from friction, a force of $\frac{1}{21}$ of 2240 lbs. = 106.666 lbs. If, now, we add to this the forces required to overcome friction, we obtain,
$$\left.\begin{array}{l} 116.666 \text{ lbs.} \\ 342.666 \text{ lbs.} \end{array}\right\} Ans.$$

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(2.) $\frac{1}{11}$ in. : 24 in. :: 13 lbs. : R, where R = resistance.
$\therefore$ R = 3432 lbs. = 1 ton 10 cwt. 2 qrs. 16 lbs.

(3.) 3.14159×24 in. $= 75.39816$ in.

Hence, $\frac{1}{9}$ in. : 75.39816 in. : : 17 lbs. : R,
$\therefore$ R = 11535.91848 = 5 tons 3 cwt., very nearly.

(4.) By the last Ex. the circumference of the circle described by power $= 2 \times 75.39816$ in. $= 150.79632$ in.

Hence, $\frac{3}{8}$ in. : 150.79632 in. : : 32 cwt. : R,
$\therefore$ R = 12867.95264 cwt. = 643 tons 8 cwt. nearly.

(5.) 1 ft. 5 in. = 17 in., 2 cwt. 1 qr. 17 lbs. = 296 lbs.
2×17 in. $\times 3.14159 = 106.81406$,

the circumference of the circle described by the handle.

$1\frac{5}{8}$ in. $\div 19 = \frac{13}{152}$ the interval between the threads.

Hence, $\frac{13}{152}$ in. : 106.81406 in. : : 296 lbs. : R.
$\therefore$ R = 335754.86809 lbs. = 149.890566 tons.

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(7.) $\frac{3}{8}$ in. $\times 3.14159 = 1.1781$ nearly.

$$\left.\begin{aligned} 1.1781^2 &= 1.38791961 \\ (\tfrac{1}{11})^2 = \tfrac{1}{121} &= .00826446 \end{aligned}\right\} \text{add}$$
$$1.39618407$$

The square root of this, viz. 1.1816 in. is length of one *wind* of the thread, $\therefore 14 \times 1.1816 = 16.5424$. *Ans.*

N. B.—The result in the Manual is incorrect.

(8.) $\frac{15}{8}$ in. $\times 3.14159 = 5.89048$ in.

$$\left.\begin{aligned} 5.89048^2 &= 34.6977546304 \\ (\tfrac{3}{8})^2 = \tfrac{9}{64} &= .140625 \end{aligned}\right\} \text{add}$$
$$34.8383796304$$

The square root of this, viz. 5.9024 in. is required length of thread.

(9.) Diameter of circle described by power = 6 in., diameter of cylinder $\frac{3}{8}$ in., and the ratio of these numbers $= 6 \div \frac{3}{8} = 16$. Also length of thread = 1.1816 in., and the ratio of this to $\frac{1}{11}$ is $1.1816 \div \frac{1}{11} = 12.9976$, $\therefore$ pressure on thread = 14 lbs. × 16 × 12.9976
= 2911.4624 lbs. = 1 ton 5 cwt. 3 qrs. 27.4624

N. B.—In the Manual, Ex. 7 has entailed its error upon this Example.

(10.) Diameter of circle described by power = 28 in., and ratio of this to diameter of cylinder $\frac{15}{8}$ in., is

$$28 \div \frac{15}{8} = \frac{224}{15};$$

also, length of one wind of thread = 5.9024, the ratio of which to $\frac{3}{8}$ is $5.9024 \div \frac{3}{8} = 15.7397$, $\therefore$ pressure on thread = 37 lbs. × $\frac{224}{15}$ × 15.7397 = 8696.7089 = 3 tons 17 cwt. 2 qrs. 16.7089 lbs. *Ans.*

(11.) In Ex. 9, distance between threads = $\frac{1}{11}$ in., and radius of circle described by power = 6 in × 3.14159 = 18.84954 in. $\therefore \frac{1}{11}$ in. : 18.84954 in. : : 14 lbs. : R,

$\therefore$ R = 2902.82916 lbs. = 1 ton 5 cwt. 3 qrs. 18.82916 lbs.

In Ex. 10, distance between threads = $\frac{3}{8}$, and radius of circle described by power

$$= 28 \text{ in.} \times 3.14159 = 87.96452 \text{ in.}$$
$$\therefore \tfrac{3}{8} : 87.96452 : : 37 \text{ lbs.} : R,$$
$$\therefore R = 8679.16597 \text{ lbs.} = 3 \text{ tons } 17 \text{ cwt. } 1 \text{ qr. } 27.16597 \text{ lbs.}$$

(12.) Fig. 37. OP′ = 5 tons, $\angle$ A = 2° = P′R′O, and we have to find OR′.

Now, sin P′R′O : sin R′P′O : : OP′ : OR′,
That is, sin 2° : cos 2° : : 5 tons : OR′,
Or, 1 : cot 2° : : 5 : OR′,

Or, $1 : 28.63625 :: 5 : OR'$,
$\therefore OR' = 143.18125$ tons $= 143$ tons 3 cwt. 2 qrs. 14 lbs.

N. B.—The result in the Manual is incorrect.

(13.) Fig. 37. Here we are to find $OV' = P'R'$.

Now, $\sin P'R'O : \sin P'OR' :: OP' : P'R'$,
That is, $.0348995 : 1 :: 5$ ton $: P'R'$
Since $\sin P'R'O = \sin 2^\circ = .0348995$, and
$\sin P'OR' = \sin 90^\circ = 1$;
$\therefore P'R' = 143$ ton 5 cwt. 1 qr. 11.8 lbs. *Ans.*

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(1.) $2^4 = 16 \therefore 16 : 1 :: 17$ ton 12 cwt. : P,
$\therefore P = 1$ ton 2 cwt. *Ans.*

(2.) $3^4 = 81 \therefore 81 : 1 :: 17$ ton 12cwt. : P.
$\therefore P = 4$ cwt. 1 qr. $10\frac{58}{81}$ lbs. *Ans.*

(3.) $2^{11} = 2048 \therefore 1 : 2048 :: 13$ lbs. : W.
$\therefore W = 26624$ lbs. $= 11$ tons 17 cwt. 2 qrs. 24 lbs. *Ans.*

(4.) $3^7 = 2187 \therefore 1 : 2187 :: 17$ lbs. : W,
$\therefore W = 37179$ lbs. $= 16$ ton 11 cwt. 3 qrs. 23 lbs. *Ans.*

(5.) Each of the two *compound* blocks in Smeaton's Pulley has 10 *single* blocks; hence, its efficiency is represented by 20; the efficiency of a Barton of the second kind, with 5 moveable pullies, is represented by $3^5 = 243$, and $243 : 20 :: 12.15 : 1$ *Ans.*

(6.)
$$2^{10} = 1024$$
$$3^{10} = 59049$$

and, $59049 \div 1024 = 57.6650390625$ Ans.

DYNAMICS.

Page 36.

(1.) 754 yards = 2262 feet.
1 hour = 3600 seconds.
2262 feet ÷ 3600 = .628333 feet. *Ans.*

(2.) 40 miles = 211200 feet.
1 hour = 3600 seconds.
211200 feet ÷ 3600 = 58.666 feet. *Ans.*

(3.) 7 hours 31 minutes = 27060 seconds.
200 miles = 1056000 feet.
1056000 ÷ 27060 = 39.0243902439. *Ans.*

(4.) 23 hours 54 minutes = 86164 seconds.

7925 miles = 41844000 feet.
∴ circumference of earth = 41844000 feet × 3.14159,
= 131456691.96 and 131456691.96 ÷ 86160
= 1525.72 feet. *Ans.*

(5.) 64 miles = 337920 feet.
337920 ÷ 1090 = $310\frac{2}{109}$ secs. = 5 m. . $10\frac{2}{109}$ secs. *Ans.*

(6.) $30\frac{1}{4}$ miles : 261 miles : : 1 hour. The fourth term of this proportion is 8 hours $37\frac{83}{121}$ minutes.

Again, $\left.\begin{array}{l}347 \text{ miles}\\ 261\end{array}\right\}$ subtract

16 minutes : 86 : : 1 hour.

The fourth term is 5 hours $22\frac{1}{2}$ minutes.

8 hours $37\frac{83}{121}$ minutes
5 ,, $22\frac{1}{2}$,,
2 ,, 27 ,, } add

16 hours $27\frac{45}{242}$ minutes. *Ans.*

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(7.) 1 hour : 4 days : : 12 knots. The fourth term is 1152 knots, and 1 hour : 3 days 7 hours : : 13 knots. The fourth term is 1027 knots.

1027 knots
1152 ,, } add

2179

Now, since 1 mile = 5280 feet, we have 5280 feet : 6076 feet : : 2179 knots. The fourth term is $2507\frac{661}{1320}$ miles. *Ans.*

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(1.) Cubic yard = 27 cubic feet, $8.9 \times 27 = 240.3$.

$\therefore 11.35 \div 240.3 = .0472326$ *Ans.*

(2.) $19.35 \times 4.17 \times .64 \times .31 = 16.0087968$
$10.51 \times 13.22 \times 1.14 \times .65 = 102.9561702$
$\therefore 16.0087968 \div 102.9561702 = .155491$ *Ans.*

(3.) By last example, mass = 102.9561702.

5 miles = 26400 feet, and 1 hour = 3600 seconds.
$\therefore 26400 \div 3600 = 7\frac{1}{3}$ feet, the velocity per second.

Hence, quantity of motion,

$= 102.9561702 \times 7\frac{1}{3} = 755.0119148$ *Ans.*

(4.) 1407 yards = 4221 feet.

$4221 \div 60 = 70.35$ feet, the velocity per second.

Also, 1 cubic foot = 1728 cubic inches, ∴ quantity of motion,

$$= 8.9 \times 1728 \times 70.35 = 1081926.72 \quad \textit{Ans.}$$

(5.) 2 yards = 6 feet.

$$\therefore \text{momentum} = 6 \times 2 \times 1.16 \times 2.716 \times 13$$
$$= 491.48736 \quad \textit{Ans.}$$

(6.) Here mass $= 1 \times 11.35 = 11.35$, and by last Ex. momentum $= 491.48$

$$\therefore \text{velocity } 491.48 \div 11.35 = 43.302 \quad \textit{Ans.}$$

(7.) Mass of the bullet of lead $= 11.35 \times .267 = 3.03045$

14 feet $\div$ 60 seconds $= \frac{7}{30}$ ft. velocity per second.

Momentum of the ball of copper

$$= 8.9 \times 13.47 \times \tfrac{7}{30} = 27.9727$$

∴ velocity of bullet of lead should be

$$= 27.9727 \div 3.03045 = 9.2305 \quad \textit{Ans.}$$

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(1.)
$$\left.\begin{array}{l} 17.14^2 = 293.7796 \\ 13.11^2 = 171.8721 \end{array}\right\} \text{add}$$
$$R^2 = 465.6517$$

$$\therefore R = 21.579 \text{ feet, very nearly.}$$

(2.) Fig. 7.
$$OB = 13.11$$
$$OA = 17.14$$
$$OC = 21.579$$

$$\therefore \cos COA = \frac{OA}{OC} = \frac{17.14}{21.579} = .79429 = \cos 37^\circ 25', \text{ and}$$

$$\therefore COB = 90^\circ - 37^\circ 25' = 52^\circ 35'.$$

(3.) Let (Fig. 7) OB be the direction of north, and OA of east.

$$4 \text{ knots} = 4 \times 6076 \text{ feet} = 24304 \text{ feet}.$$

1 hour = 3600 seconds $\therefore$ velocity of ship per second $= 24304 \div 3600 = 6.7511$ feet $=$ OB.

$$\left.\begin{aligned} 6.7511^2 &= 45.57735121 \\ OA^2 = 10^2 &= 100. \end{aligned}\right\}\text{add}$$

$$OC^2 = 145.57735121$$

$$\therefore OC = 12.0655 \text{ feet. } \textit{Ans.}$$

(4.) Fig. 7. $\cos COB = \frac{OB}{OC} = \frac{6.7511}{12.0655}$

$$= .5595375 = \cos 55^\circ 59'. \quad \textit{Ans.}$$

(5.) $$\left.\begin{aligned} 36.14^2 &= 1306.0996 \\ 14.31^2 &= 204.7761 \end{aligned}\right\}\text{subtract}$$

$$1101.3235$$

The square root of this = 33.1851 feet. *Ans.*

(6.) $$\cos 22^\circ = .92718.$$

$$\left.\begin{aligned} 2 \times 14.37 \times 19.22 \times .92718 &= 512.158284504 \\ 14.37^2 &= 206.4969 \\ 19.22^2 &= 369.4084 \end{aligned}\right\}\text{add}$$

$$R^2 = 1088.063584504$$

$$\therefore R = 32.9858 \text{ feet. } \textit{Ans.}$$

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(7.) Fig. 5.

$$OA = 19.22$$
$$OB = 14.37$$
$$OC = 33 \textit{ nearly.}$$
$$AOB = 22^\circ, \text{ and } \sin 22^\circ = .37460$$

Now, OC : OA : : (sin OAC =) sin AOB : (sin ACO =) sin BOC.

That is, $33 : 19.22 : : .3746 : \sin BOC$.
$\therefore \sin BOC = .2181761 = \sin 12^\circ\ 36'$ one angle,
and $22^\circ - 12^\circ\ 36' = 9^\circ\ 24'$, the other angle.

(8.)

$$\cos 41^\circ = .75471$$
$$14.62 \text{ yards} = 43.86 \text{ feet.}$$
$$43.86 \div 60 \text{ seconds} = .731 \text{ ft. per second.}$$

$$\left.\begin{array}{rl} 2 \times 13.61 \times .731 \times .75471 = & 15.0170837322 \\ 13.61^2 = & 185.2321 \\ .731^2 = & .534361 \end{array}\right\} \text{add}$$

$$R^2 = 200.7835447322$$

$\therefore R = 14.17$ feet, very nearly. *Ans.*

(9.) Fig. 5.

$$OA = 13.61$$
$$OB = .731 \quad \text{(See last Ex.)}$$
$$OC = 14.17$$
$$AOB = 41^\circ$$

$\therefore \sin AOB = \sin 41^\circ = .65606.$

Now, OC : (CA =) OB : : (sin OAC =) sin AOB : sin AOC.
That is, $14.17 : .731 : : .65606 : \sin AOC$.
$\therefore \sin AOC = .0338447 = \sin 1^\circ\ 56'\ 22''$,
and $\therefore BOC = 41^\circ - 1^\circ\ 56'\ 22'' = 39^\circ\ 3'\ 38''$.

(10.) Fig. 5. (Key.)

Let O be the point from which the boat starts, OA = *2.3 in* direction of stream, OB = 5, and $\angle BOG = 87^\circ$

OD being $\perp$ to the bank of the river. Complete the parallelogram AOBC, and OC will represent the *direction* of the boat, and we are to find DF.

Now, $\cos AOB = -\cos BOG = -\cos 87° = -.052336.$

$$\left.\begin{aligned} 2.3^2 &= 5.29 \\ 5^2 &= 25. \end{aligned}\right\} \text{add}$$

$$30.29$$

$$2 \times 5 \times 2.3 \times .052336 = 1.203728$$

Subtracting this from the last sum, we have,

$$OC^2 = 29.086272$$
$$\therefore OC = 5.393$$

Again,

OC : (AC =) OB : : (sin CAO =) sin BOG : sin AOC.

That is, 5.393 : 5 : : sin 87° : sin AOC.
Or, 5.393 : 5 : : .9986295 : sin AOC.
$\therefore$ sin AOC = .9258571 = sin 67° 48′, and
$\therefore$ DOF = 90° − 67° 48′ = 22° 12′.
$\therefore$ tan DOF = .4080924.

Again, DO = $1\frac{1}{4}$ mile = 2200 yards.
$\therefore$ DF = DO × tan DOF,
= 2200 × .4080924 = 897.80328 yards. *Ans.*

(11.) Fig. 6. (Key.)

Here,
$$OA = 2.3$$
$$OB = 5.$$
$$\angle AOB = 87°$$

and we are to find DF.

Now, since $OC^2 = OA^2 + OB^2 + 2\,OA \times OB \cos AOB$, *we have by last* Ex.,

$$\left.\begin{aligned} OA^2 &= 5.29 \\ OB^2 &= 25. \\ 2\,OA \times OB \cos AOB &= 1.203728 \end{aligned}\right\} \text{add}$$

$$OC^2 = 31.493728$$

$$\therefore OC = 5.612, \text{ very nearly.}$$

Again,

$$OC : (AC =)\ OB :: (\sin CAO =) \sin AOB : \sin AOC.$$

$$\text{That is, } 5.612 : 5 :: \sin 87^\circ : \sin AOC.$$
$$\text{Or, } 5.612 : 5 :: .9986295 : \sin AOC.$$
$$\therefore \sin AOC = .8897268 = \sin 62^\circ\ 50'$$
$$\therefore DOF = 27^\circ\ 10', \text{ and } \tan 27^\circ\ 10' = .5131949$$

$$\text{Also, } DO = 2200 \text{ yards (by last Ex.)}$$
$$\therefore DF = 2200 \times .5131949 = 1129.02878 \text{ yards.} \quad \textit{Ans.}$$

(12.) Fig. 7 (Key).

N. E. S. W. denote the 4 cardinal points respectively, and $\therefore \angle AOE = 22^\circ\ 30'$ and $BOE = 11^\circ\ 15'$.

$$OA = 5 \text{ knots, } OB = 3.$$

And completing the ∥m OACB, OC will denote the ship's real motion in *direction* and *magnitude.*

$$\text{Now, } \angle AOB = 22^\circ\ 30' + 11^\circ\ 15' = 33^\circ 45',$$
$$\text{and } \sin 33^\circ\ 45' = .5555702$$
$$\cos 33^\circ\ 45' = .8314696$$

$$\left.\begin{aligned} 2 \times 5 \times 3 \times .8314696 &= 24.944088 \\ 5^2 &= 25. \\ 3^2 &= 9. \end{aligned}\right\} \text{add}$$

$$OC^2 = 58.944088$$

$$\therefore OC = 7.677 \text{ knots.} \quad \textit{Ans.}$$

(13.) See fig. in last Ex.

$$OC : OA :: (\sin OAC =) \sin AOB : (\sin OCA =) \sin COB.$$

That is, $7.677 : 5 :: .5555702 : \sin COB$.

$\therefore \sin COB = .3618406 = \sin 21^\circ\ 12'$,

From which, subtracting $BOE = 11^\circ\ 15'$, we get $COE = 9^\circ\ 57'$, and therefore $NOC = 90^\circ - 9^\circ\ 57' = 80^\circ\ 3'$. *Ans.*

(14.) Fig. 8 (Key).

Let OA in direction of stream = 2.3, and OB in direction in which boat should be rowed = 5, then completing the $\|^m$ OACB, its diagonal OC should be $\perp$ to OA, we are therefore to find $\angle$ BOG = CAO.

Now, since $\angle$ COA is right, we have

$$\cos CAO = \frac{AO}{AC} = \frac{2.3}{5} = .46$$
$$= \cos 62^\circ\ 36'\ 47''. \quad \textit{Ans.}$$

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(1.) A force = weight of one grain would produce a velocity of 32.1948 feet per second in the quantity of matter contained in 1 grain of water, therefore it would produce a velocity of 1 foot per second in the quantity of matter contained in 32.1948 grs. The unit of volume will therefore be the fraction of a cubic inch contained in 32.1948 grs. of water, which is found thus—

252.458 grs. : 32.1948 grs. : : 1 inch.

The fourth term is .1275 in. *Ans.*

(2.) By reasoning the same as in the last it will appear that the unit of mass will be 32.1948 tons, reduced to cubic yards, which will be done thus—

$$\begin{array}{rl} 252.458 \text{ grs.} : & 32.1948 \text{ tons} : : 1 \text{ inch.} \\ & 2240 \\ \hline \text{Product} = & 72116.352 \text{ lbs.} \\ & 7000 \\ \hline \text{Product} & 504814464.000 \text{ grs.} \end{array}$$

The last product, divided by 252.458, gives for quotient 1999597.810 in., 1999597.810 in. ÷ 1728 = 1157.174 feet, 1157.174 ÷ 27 = 42.858 yards. *Ans.*

(3.) Here we have unit of

$$F = \frac{\text{weight of cubic foot of water}}{32.1948},$$

But weight of a cubic foot of water

$$= 252.458 \text{ grs.} \times 1728 = 436247.424 \text{ grs.}$$

$$\therefore \text{ unit of } F = \frac{436247.424}{32.1948} = 13550.244 \text{ grs.}$$

And dividing this by 7000 we obtain 1.935749 lbs. *Ans.*

(4.) A cubic inch of water = 252.458 grs., ∴ a cubic inch of lead = 252.458 × 11.35 = 2865.3983 grs.

$$\therefore \text{ unit of } F = \frac{2865.3983}{32.1948} = 89.0018978 \text{ grs.} \quad \textit{Ans.}$$

(5.) The weight of the body would produce in 1 second a velocity of 32.1948 feet,

$$\therefore .317 \text{ ft.} : 32.1948 \text{ ft.} : : 3 \text{ lbs.} : \text{weight of body } (F).$$
$$\therefore F = 304.68265$$

Which is reduced to cubic feet of water thus—

$$\begin{array}{rl} 252.458 \text{ grs.} : & 304.68265 \text{ lbs.} : : 1 \text{ inch.} \\ & 7000 \\ \hline \text{Product} = & 2132778.55 \end{array}$$

Dividing this by 252.458, we find 8448.052 inches, and 8448.052 in. $\div$ 1728 = 4.8889 feet. *Ans.*

(6.) 252.458 grs. = one cubic inch of water, $\therefore$ 252.458 $\times$ 1728 = 436247.424 grs. = one cubic foot of water.

$$13.16 \text{ lbs.} = 92120 \text{ grs.}$$

Now, 436247.424 grs. would produce in a cubic foot of water a velocity of 32.1948 feet in one second, hence, we find the velocity which 13.16 lbs. would generate in one foot of water in one second, thus—

$$436247.424 \text{ grs.} : 92120 \text{ grs.} :: 32.1948 \text{ feet.}$$

The fourth term is 6.798 feet, and as the velocities generated in *the same volume* by the *same force* in one second, must be *inversely* as the specific gravities, we have (the sp. gr. gravity of water being unity),

$$4.16 \text{ ft.} : 6.798 \text{ ft.} :: 1 : \text{sp. gr. of the matter,}$$
$$\therefore \text{sp. gr. of the matter} = 1.6341 \quad \textit{Ans.}$$

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(7.) A cubic inch of water = 252.458 grs., $\therefore$ cubic inch of gold = 252.458 $\times$ 19.35 = 4885.0623 grs.

Now, 4885.0623 grs. would generate in a cubic inch of gold a velocity of 32.1948 feet in one second; and since the velocity varies as the force, we have

$$32.1948 \text{ ft.} : 42.31 \text{ ft.} :: 4885.0623 \text{ grs.} : \text{required force,}$$

$$\therefore \text{required force} = 6419.887 \text{ grs.} \quad \text{Ans.}$$

(8.) 17 lbs. = 119000 grs.

One cubic foot of water = 436247.424 grs. (See Ex. 6, p. 44.) Hence, we find the velocity (V) which 17 lbs. would generate in a cubic foot of water in one second, thus—

$$436247.424 : 119000 :: 32.1948 : V, \therefore V = 8.782 \text{ ft.}$$

Hence, 14 ft. : 8.782 ft. : : 1 sp. gr. of water : sp. gr. of first matter, ∴ sp. gr. of first matter = .627.

Again, one cubic inch of water = 252.458 grs., and 1 ounce $= \frac{7000}{16} = 437.5$ grs. Hence, we find the velocity (v) which 1 ounce would generate in a cubic inch of water, thus—

$$252.458 \text{ grs.} : 437.5 \text{ grs.} :: 32.1948 \text{ feet} : v.$$
$$\therefore v = 55.792 \text{ feet.}$$

Hence, 11 : 55.792 : : 1 : sp. gr. of second matter.
∴ sp. gr. of second matter = 5.072.

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(1.) 3 minutes = 180 seconds.

$$\therefore f = \frac{v}{t} = \frac{14}{180} = .0777 \text{ ft.} \quad \textit{Ans.}$$

(2.) 4 miles = 21120 ft., and 1 hour = 3600 seconds.

$$\therefore f = \frac{21120}{13 \times 3600} = .451 \text{ ft. per second.}$$

(3.) 3 minutes = 180 seconds.

$$\therefore v = ft = 7 \times 180 = 1260 \text{ ft. per second.}$$

(4.) $v = ft = 32.195 \times 5 = 160.975$ ft.

(5.) $t = \frac{v}{f} = \frac{100}{11.16} = 8.96$ seconds.

(6.) $t = \frac{v}{f} = \frac{375}{32.195} = 11.647$ seconds.

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(1.) $$s = \tfrac{1}{2}ft^2.$$

$$\therefore = \frac{2s}{t^2} = \frac{10}{9} = 1.111 \text{ feet.}$$

(2.) 300 yards = 900 feet.

$$f = \frac{2s}{t^2} = \frac{1800}{100} = 18 \text{ feet.}$$

(3.) 3 minutes = 180 seconds.

$$s = \tfrac{1}{2}ft^2 = \frac{11 \times 180^2}{2} = \frac{11 \times 32400}{2}$$

$$= 178200 \text{ feet.}$$

(4.) $s = \frac{1}{2}ft^2 = \frac{32.19 \times 49}{2} = 788.655$ feet.

(5.) 137 yards = 411 feet.

$$t^2 = \frac{2s}{f} = \frac{822}{12.17} = 67.543138.$$

$$\therefore t = 8.218 \text{ seconds.}$$

(6.) $s = \frac{1}{2}ft^2 = \frac{32.18 \times 121}{2} = 1946.89$ feet.

Page 50.

1.) 100 yards = 300 feet.

$$v^2 = 2fs.$$

$$\therefore f = \frac{v^2}{2s} = \frac{35^2}{600} = \frac{1225}{600} = 2.04166 \text{ feet.}$$

(2.) 1142 yards = 3426 feet.

$$\therefore \frac{3426}{60} = 57.1 \text{ ft. per second} = v.$$

$$1 \text{ mile} = 5280 \text{ ft.} = s.$$

$$\therefore f = \frac{v^2}{2s} = \frac{3260.41}{10560} = .30875 \text{ feet.}$$

(3.) $v^2 = 2fs = 2 \times 13 \times 1100 = 28600$

$$\therefore v = 169.115 \text{ feet.}$$

(4.) $v^2 = 2fs = 2 \times 32.19 \times 467$

$$= 30065.46$$

$$\therefore v = 173.3939 \text{ feet.}$$

(5.) $s = \frac{v^2}{2f} = \frac{75^2}{64.38} = \frac{5625}{64.38} = 87.37$ feet.

(6.) 314 yards = 942 ft., and $\frac{942}{60} = 15.7$ ft. per second.

$$\therefore s = \frac{v^2}{2f} = \frac{411^2}{31.4} = 5379.65 \text{ feet.}$$

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(1.) In this and the three following examples $g = 32$ feet.

The ninth term of the series, 1, 3, 5, 7, 9, 11, 13, 15, 17, &c., is 17 (found either by actually continuing the series to the ninth term, or by the common rule for finding any term of an arithmetical progression when the first term and the common difference are given; we might also have found the term thus—

$$9^2 - 8^2 = 81 - 64 = 17)$$
$$\therefore 17 \times 16 = 272 \text{ feet. } \textit{Ans.}$$

(2.) $s = \frac{1}{2}gt^2 = 16 \times 81 = 1296$ feet. *Ans.*

(3.) $7^2 - 4^2 = 49 - 16 = 33$
$\therefore$ required space $= 33 \times 16 = 528$ feet. *Ans.*

(4.) $11^2 - 3^2 = 121 - 9 = 112$
$\therefore$ required space $= 112 \times 16 = 1792$ feet. *Ans.*

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(4.)
$$g = 32.1412$$
$$\therefore \tfrac{1}{2}g = 16.0706$$
$$t = 3.151$$
$$\therefore t^2 = 9.928801$$
$$\therefore s = \tfrac{1}{2}gt^2 = 16.0706 \times 9.928801$$
$$= 159.5617893506 \text{ feet. } \textit{Ans.}$$

(5.)
$$g = 32.16 \text{ feet.}$$
$$s = 316 \text{ feet.}$$
$$\text{and, } s = \tfrac{1}{2}gt^2.$$

$$\therefore t^2 = \frac{2s}{g} = \frac{632}{32.16} = 19.6517$$

$$\therefore t = 4.43 \text{ seconds.} \quad \textit{Ans.}$$

(6.)
$$g = 32.182$$
$$v = gt = 32.182 \times 9 = 289.638 \text{ feet.} \quad \textit{Ans.}$$

(7.)
$$g = 32.0927$$
$$v^2 = 2gs = 2 \times 32.0927 \times 216.171$$
$$= 13875.0221034$$
$$\therefore v = 117.7922 \text{ feet.} \quad \textit{Ans.}$$

(8.)
$$t^2 = \frac{2s}{g} = \frac{432.342}{32.0927} = 13.4716$$
$$\therefore t = 3.67 \text{ seconds.} \quad \textit{Ans.}$$

(9.)
$$g = 32.1403$$
$$v = 71.3$$
$$v^2 = 2gs.$$

$$\therefore s = \frac{v^2}{2g} = \frac{5083.69}{64.2806} = 79.0859 \text{ feet.}$$

(10.)
$$v = gt.$$
$$\therefore t = \frac{v}{g} = \frac{71.3}{32.1403} = 2.218 \text{ seconds.} \quad \textit{Ans.}$$

(11.)
$$g = 32.1691;\ v = 131.17$$
$$t = \frac{v}{g} = \frac{131.17}{32.1691} = 4.0775 \text{ seconds.}$$

(12.)
$$g = 32.0917;\ s = 200$$
$$s = \tfrac{1}{2}gt^2$$
$$\therefore t^2 = \frac{2s}{g} = \frac{400}{32.0917} = 12.4642$$
$$\therefore t = 3.53 \text{ seconds.} \quad \textit{Ans.}$$

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(4.)
$$g = 32.1691$$
$$\sin i = \frac{1}{65}$$
$$s = 427$$

$$\text{and } f = g \sin i = \frac{32.1691}{65} = .49491$$

$$\therefore v^2 = 2fs = 2 \times 427 \times .49491$$
$$= 422.65314$$
$$\therefore v = 20.558 \text{ feet. } \textit{Ans.}$$

(5.)
$$s = \tfrac{1}{2}ft^2$$

$$\therefore t^2 = \frac{2s}{f} = \frac{854}{.49491} = 1725.5662$$

$$\therefore t = 41.54 \text{ seconds. } \textit{Ans.}$$

(6.)
$$s = \tfrac{1}{2}ft^2 = \tfrac{1}{2} \times .49491 \times 9$$
$$= 2.22709 \text{ feet. } \textit{Ans.}$$

(7.)
$$g = 32.19$$
$$2240 \text{ lbs.} - 7 \text{ lbs.} = 2233, \text{ and } \frac{2233}{2240} = \frac{319}{320}$$

$\therefore$ the accelerating force of gravity is diminished in the ratio of 319 : 320 by the resistance from friction and the air.

$$\sin i = \frac{31}{2164}$$

$$\therefore f = 32.19 \times \frac{31}{2164} \times \frac{319}{320} = .45969 \text{ feet.}$$

But it is not necessary to use this value of f *here* since the velocity acquired in falling down the vertical height,

31 feet, will be the same as that acquired by running down the inclined plane, g being diminished in the ratio of $319 : 320$, which will then become $\frac{319 \times 32.19}{320}$.

$$\therefore v^2 = 2 \times \frac{319 \times 32.19}{320} \times 31 = 1989.54318$$

$$\therefore v = 44.6 \text{ feet.} \quad \textit{Ans.}$$

(8.)
$$s = 1000 \text{ feet.}$$
$$f = .45969 \quad \text{(By last Example.)}$$
$$s = \tfrac{1}{2}ft^2$$
$$\therefore t^2 = \frac{2s}{f} = \frac{2000}{.45969} = 4350.7581$$
$$\therefore t = 65.96 \text{ seconds.} \quad \textit{Ans.}$$

(9.)
$$g = 32.2435$$
$$\sin i = \sin 14^\circ = .24192$$
$$\therefore f = g \sin i = 32.2435 \times .24192$$
$$= 7.80034752$$
$$\therefore s = \tfrac{1}{2}ft^2 = 3.90017376 \times 25$$
$$= 97.504344 \text{ feet.} \quad \textit{Ans.}$$

(10.)
$$g = 32.093$$
$$\therefore f = 32.093 \times .24192 = 7.76393856$$
$$\therefore s = \tfrac{1}{2}ft^2 = 3.88196928 \times 25$$
$$= 97.049232 \text{ feet.} \quad \textit{Ans.}$$

(11.) $s = 100$, $f = 7.8$ very nearly (by Ex. 9).
$$v^2 = 2fs = 2 \times 100 \times 7.8 = 1560$$
$$\therefore v = 39.5 \text{ feet.} \quad \textit{Ans.}$$

(12.) $v = ft = 7.8 \times 11 = 85.8$ feet. *Ans.*

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(1.) 300 yards = 900 feet.

$$\therefore v = \frac{900}{60} = 15 \text{ feet per second.}$$

$$\therefore f = \frac{v^2}{r} = \frac{225}{16} = 14.0625 \text{ feet. } \textit{Ans.}$$

(2.) $r = 17$ yards $= 51$ feet, $v = 361$ feet.

$$\therefore f = \frac{v^2}{r} = \frac{130321}{51} = 2555.3137 \text{ feet. } \textit{Ans.}$$

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(4.) Let W = weight of body, F = *statical* measure of centrifugal force, f = *dynamical* measure of centrifugal force,

$$\therefore W = 1000 \text{ lbs.}$$
$$F = 450 \text{ lbs.}$$
$$r = 100 \text{ feet.}$$

Now, centrifugal force in lbs. : weight of body in lbs. : : centrifugal force in feet : gravity in feet.

That is, $F : W :: f : g$

$$\text{Or, } 450 \text{ lbs.} : 1000 \text{ lbs.} :: \frac{v^2}{r} : 32.1948 \text{ feet.}$$

$$\text{Or, } 45 : 100 :: \frac{v^2}{r} : 32.1948$$

$$\text{Or, } 45 : 100 :: \frac{v^2}{100} : 32.1948$$

$$\therefore v^2 = 45 \times 32.1948 = 1448.766$$
$$\text{and } \therefore v = 38.06 \text{ feet. } \textit{Ans.}$$

(5.) 30 miles = 158400 feet.
1 hour = 3600 seconds.

$$\therefore v = \frac{158400}{3600} = 44 \text{ feet.}$$

$$r = 460 \text{ yards} = 1380 \text{ feet.}$$

$$\therefore f = \frac{v^2}{r} = \frac{44^2}{1380} = \frac{1936}{1380} = 1.4029 \text{ feet } \textit{very nearly.}$$

Also, W = 7.21 tons.
Now, $g : f :: \text{W} : \text{F}$
That is, 32.1948 feet : 1.4029 feet.
:: 7.21 tons : F
$\therefore$ F = .314 tons. *Ans.*

(6.) Let v = velocity of body weighing 13 lbs.
v' = ditto 16 lbs.
and F = centrifugal force of *either* body in lbs.

W = 13 lbs.
W′ = 16 lbs.
r = 11 feet.
r' = 15 feet.

Now, $\text{W} : \text{F} :: g : f.$

That is, $13 \text{ lbs.} : \text{F} :: g : \frac{v^2}{r};$

Or, $13 \text{ lbs.} : \text{F} :: g : \frac{v^2}{11};$

Or, $13 \text{ lbs.} : \text{F} :: 11\, g : v^2;$

$$\therefore v^2 = \frac{11 \text{ F} \times g}{13}.$$

Again, $\text{W}' : \text{F} :: g : f';$

That is, $16 \text{ lbs.} : \text{F} :: g : \frac{v'^2}{r'};$

Or, $16 \text{ lbs.} : \text{F} :: g : \frac{v'^2}{15};$

Or, 16 lbs. : F : : 15 g : v'^2;

$$\therefore v'^2 = \frac{15 \text{ F} \times g}{16}.$$

Hence, $$\frac{v^2}{v'^2} = \frac{11 \text{ F} \times g}{13} \div \frac{15 \text{ F} \times g}{16}$$

$$= \frac{11}{13} \div \frac{15}{16} = \frac{11}{13} \times \frac{16}{15} = \frac{176}{195} = .902564$$

And $\therefore$ by extracting the square root we obtain

$$\frac{v}{v'} = .9503 \ \textit{Ans.} \quad \text{And } \frac{20}{21} = .9523$$

(7.) $T = 16$ seconds.
$r = 11$ feet.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 3.14159^2 \times 11}{16^2}$$

$$= \frac{4 \times 3.14159^2 \times 11}{256} = \frac{9.8695877 \times 11}{64}$$

$$= \frac{108.5654647}{64} = 1.6963353 \text{ feet.} \quad \textit{Ans.}$$

(8.) $r = 11$ feet.
11 minutes = 660 seconds.
$\therefore 61^\circ : 360^\circ : : 660$ seconds : T.
$\therefore T = 3895.082$ seconds, *very nearly.*

Hence, $$f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 11}{3895.082^2}$$

$$= \frac{9.8695877 \times 11}{1947.541^2} = \frac{108.5654647}{3792915.946681}$$

$$= .0000286 \text{ feet.} \quad \textit{Ans.}$$

(9.) $r = 11$ yards $= 33$ feet.
$T = 4$ minutes $= 240$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 33}{240^2}$$

$$= \frac{9.8695877 \times 33}{120^2} = \frac{9.8695877 \times 33}{14400}$$

$$= \frac{9.8695877 \times 11}{4800} = \frac{108.5654647}{4800}$$

$= .0226178$ feet. *Ans.*

(10.) $r = 100$ feet.
$f = 146$ feet.

But we have also by formula (13)

$$f = \frac{4\pi^2 r}{T^2}$$

$$\therefore T^2 = \frac{4\pi^2 r}{f} = \frac{4 \times 9.8695877 \times 100}{146}$$

$$= \frac{1973.91754}{73} = 27.04, \textit{ very nearly.}$$

$\therefore T = 5.2$ seconds. *Ans.*

(11.) $F = 131$ oz.
$r = 100$ feet.
$T = 1$ hour $= 3600$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 100}{3600^2}$$

$$= \frac{9.8695877}{180^2} = \frac{9.8695877}{32400}$$

$= .0003046017$ feet.

Now, $f : g :: F : W$;
That is, .000304617 feet : 32.1948 feet : : 131 oz. : W ;
$\therefore$ W = 13845316.5 oz.
= 865332.2 lbs. = 386.3 tons. *Ans.*

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(3.) $\cos 23^\circ = .9205$
$\sin 23^\circ = .39073$
$\therefore f = .11126 \cos^2 l = .11126 \times .9205^2$
$= .11126 \times .8473225$
$= .0942731 0135$ feet. *Ans.*

(4.) $\phi = .11126 \cos l \sin l$
$= .11126 \times .9205 \times .39073$
$= .0400165$ feet. *Ans.*

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(5.) $\sin 53^\circ\ 21' = .8022969$
$\cos 53^\circ\ 21' = .5969252$
$\therefore \cos^2 53^\circ\ 21' = .35631969$
and $\cos 53^\circ\ 21' \times \sin 53^\circ\ 21' = .47891123$
$\therefore f = .11126 \times .35631969 = .03964412$ feet,
and $\phi = .11126 \times .47891123 = .05328366$ feet.

(6.) $r = 441000$ miles $= 2328480000$ feet.
T = 607 hours 48 minutes = 2188080 seconds.
$\therefore$ by equation (13) we have (recollecting that $\pi = 3.14159$ and $\therefore \pi^2 = 9.8695877$),

$$f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 2328480000}{2188080^2}$$

$$= \frac{986.95877 \times 23848}{109404^2} = \frac{229811375.67696}{11969235216}$$

$= .01920017$ feet. *Ans.*

(7.) $r = 1570$ miles $= 8289600$ feet.
$T = 24$ hours 5 minutes $= 86700$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 8289600}{86700^2}$$

$$= \frac{.098695877 \times 331584}{751689}$$

$$= \frac{32725.973679}{751689} = .043536 \text{ feet. } \textit{Ans.}$$

(8.) $r = 3900$ miles $= 20592000$ feet.
$T = 23$ hours 21 minutes $= 84060$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 20592000}{84060^2}$$

$$= \frac{98.695877 \times 20592}{4203^2} =$$

$$\frac{2032345.499184}{17665209} = .1150479 \text{ feet. } \textit{Ans.}$$

(9.) $r = 2050$ miles $= 10824000$ feet.
$T = 24$ hours 37 minutes $= 88620$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 10824000}{88620^2}$$

$$= \frac{98.695877 \times 10824}{4431^2} = \frac{1068284.172648}{19633761}$$

$= .054415$ feet. *Ans.*

(10.) $r = 43500$ miles $= 229680000$ feet.
$T = 9$ hours 56 minutes $= 35760$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877281 \times 229680000}{35760^2}$$

$$= \frac{986.95877281 \times 22968}{3196944}$$

$$= \frac{986.95877281 \times 2871}{399618} = \frac{2833558.63673751}{399618}$$

$$= 7.090668 \text{ feet. } \textit{Ans.}$$

(11.) $r = 39580$ miles $= 208982400$ feet.
$T = 10$ hours 29 minutes $= 37740$ seconds.

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 208982400}{37740^2}$$

$$= \frac{9.8695877 \times 2089824}{1887^2}$$

$$= \frac{20625701.2455648}{3560769} = 5.79248 \text{ feet. } \textit{Ans.}$$

APPENDIX.

N. B.—As proofs of the Rules in the Appendix to the Manual may not suggest themselves to every reader, they are here supplied.

RULE I.

$$\sin(\theta+\epsilon)=\sin\theta\cos\epsilon+\cos\theta\sin\epsilon.$$

If in this formula we suppose θ to denote degrees, and ϵ minutes, $\cos\epsilon$ will be very nearly $= 1$, and hence the foundation of Rule I.

RULE II.

The proof is obvious from the same formula.

RULES III. AND IV.

$$\cos(\theta+\epsilon)=\cos\theta\cos\epsilon-\sin\theta\sin\epsilon=\cos\theta-\sin\theta\sin\epsilon \text{ very nearly.}$$

RULES V AND VI.

$$\tan(\theta+\epsilon)=\frac{\tan\theta+\tan\epsilon}{1-\tan\theta\tan\epsilon}=$$

(by dividing the numerator by the denominator.)

$$\tan\theta+\frac{\tan\epsilon\,(1+\tan^2\theta)}{1-\tan\theta\tan\epsilon}$$

$$=\tan\theta+\frac{2\tan\epsilon}{2\cos^2\theta\,(1-\tan\theta\tan\epsilon)}$$

$$=\tan\theta+\frac{2\sin\epsilon}{(1+\cos 2\theta)\cos\epsilon\,(1-\tan\theta\tan\epsilon)}.$$

$$= \tan\theta + \frac{2 \sin \epsilon}{1 + \cos 2\theta} \text{ since } \cos\epsilon = 1 \text{ nearly,}$$

And $\tan\theta \tan\epsilon$ may be neglected, as ϵ is very small.

RULES 1° and 2°.—When the angle is very small its circular measure, sine, and tangent, are very nearly equal,

$$\therefore \tan(\theta+\epsilon) = \frac{\tan\theta + \tan\epsilon}{1 - \tan\theta\tan\epsilon} = \frac{\tan\theta + \sin\epsilon}{1 - \tan\theta \sin\epsilon}$$

approximately, since ϵ is small.

RULE I.

(2.) $\left.\begin{array}{r} \sin 34° = .55918 \\ \text{First five figs.} = .00651 \end{array}\right\}$ add $\quad$ $\cos 34° = .82904$, $\sin 27' = .00785$

$\therefore \sin 34° \, 27' = .56569$ $\quad$ Product $= .0065079 64$

(3.) $\left.\begin{array}{r} \sin 54° = .80901 \\ \text{First five figs.} = .00102 \end{array}\right\}$ add $\quad$ $\cos 54° = .58778$, $\sin 6' = .00174$

$\sin 54° \, 6' = .81003$ $\quad$ Product $= .0010227372$

(4.) $\left.\begin{array}{r} \sin 69° = .93358 \\ \text{First five figs.} = .00344 \end{array}\right\}$ add $\quad$ $\cos 69° = .35837$, $\sin 33' = .00960$

$\therefore \sin 69° \, 33' = .93702$ $\quad$ Product $= .003440352$

RULE II.

(2.) Given sine $= .78231$
$\sin 51° = .77714$, the next lower in Table II.

$.00517$ = difference.

$\cos 51° = .62932$ and $.00517 \div .62932 = .00821$

which (Table IV.) most nearly corresponds to $28'$

$\therefore$ required $\angle = 51° \, 28$

(3.) Given sine $= .31420$
$\sin 18° = .30901$

cos 18° = .95105)$.00519$ = difference.

$.00545$ = Quot. = sin 19′ nearly, by (Table IV.)

$\therefore$ required $\angle = 18° \, 19'$

(4.) Given sine $= .80000$
$\sin 53° = .79863$

$\cos 53° = .60181).00137 =$ difference.

$.00227 =$ Quot. $= \sin 8'$

$\therefore$ required $\angle = 53° \; 8'$

RULE III.

(2.)
$$\left.\begin{array}{r} \cos 7° = .99254 \\ \text{First five figs.} = .00095 \end{array}\right\} \text{subtract} \qquad \begin{array}{l} \sin 7° = .12187 \\ \sin 27' = .00785 \end{array}$$

$\therefore \cos 7° \; 27' = .99159$ Product $= .0009566795$

(3.)
$$\left.\begin{array}{r} \cos 53° = .60181 \\ \text{First five figs.} = .00464 \end{array}\right\} \text{subtract} \qquad \begin{array}{l} \sin 53° = .79863 \\ \sin 20' = .00582 \end{array}$$

$\therefore \cos 53° \; 20' = .59717$ Product $= .0046480266$

(4.)
$$\left.\begin{array}{r} \cos 32° = .84805 \\ \text{First five figs.} = .00601 \end{array}\right\} \text{subtract} \qquad \begin{array}{l} \sin 32° = .52992 \\ \sin 39' = .01134 \end{array}$$

$\therefore \cos 32° \; 39' = .84204$ Product $= .0060092928$

RULE IV.

(2.) $\cos 36° = .80901$, the next higher in the Tables.
given cosine $= .80000$

$\sin 36° = .58778).00901 =$ difference.

$.01532 =$ Quot. $= \sin 52'$ (by Table IV.)

$\therefore$ required angle $= 36° \; 52'$

(3.) $\cos 61° = .48481$, the next higher in the Tables.
given cosine $= .47320$

$\sin 61° = .87462).01161 =$ difference.
$.01327 =$ quot. $= \sin 46'$

$\therefore$ required angle $= 61° \; 46'$

(4.) $\cos 69° = .35837$, the next higher in the Table
given cosine $= .34955$

$\sin 69° = .93358).00882 =$ difference.

$.00944 =$ Quot. $= \sin 32'$

$\therefore$ required angle $= 69°\ 32'$

RULE V.

(2.) $2 \times 57° = 114°$

Now $\cos 114° = -\cos 66°$ (its supplement)

$$\left.\begin{array}{r} 1.00000 \\ \cos 66° = \quad .40673 \end{array}\right\} \text{subtract, since } \cos 114° \text{ is } \textit{negati}$$

$2)\ .59027$

Half-sum $= .29663)\ .00901 = \sin 31'$

$.03037 =$ Quot.

$$\left.\begin{array}{r} \tan 57° = 1.53986 \\ .03037 \end{array}\right\} \text{add}$$

$\therefore \tan 57°\ 31' = 1.57023$

(3.) $2 \times 49° = 98°$

$\therefore \cos 98° = -\cos 82°$

$$\left.\begin{array}{r} 1.00000 \\ \cos 82° = \quad .13917 \end{array}\right\} \text{subtract}$$

$2)\ .86083$

$.43041)\ .00989 = \sin 34'$

$.02297 =$ quot.

$$\left.\begin{array}{r} \tan 49° = 1.15037 \\ .02297 \end{array}\right\} \text{add}$$

$\therefore \tan 49°\ 34' = 1.17334$

(4.) $2 \times 19° = 38°$

$$\left.\begin{array}{r} \cos 38° = .78801 \\ 1.00000 \end{array}\right\} \text{add}$$

$2)\ 1.78801$

$.89400)\ .00756 = \sin 26'$

$.00845 =$ Quot.

$$\left.\begin{array}{r}\tan 19^\circ = .34432 \\ .00845\end{array}\right\}\text{ add}$$

$\therefore \tan 19^\circ\ 26' = .35277$

RULE VI.

(2) Given $\tan = .87245$

$\tan 41^\circ = .86928$ next lower in the Tables.

Diff. $= .00317$

$2 \times 41^\circ = 82^\circ$

$$\left.\begin{array}{r}\cos 82^\circ = \ .13917 \\ 1.00000\end{array}\right\}\text{ add}$$

$2)\ 1.13917$

$.56958$

Diff. $= .00317$

Product $= .00180, 55686 = \sin 6'$

$\therefore \tan 41^\circ\ 6' = .87245$

(3.) Given $\tan = 1.14376$

$\tan 48^\circ = 1.11061$

Diff. $= .03315$

Now, $\cos 96^\circ = -\cos 84^\circ$

$$\left.\begin{array}{r}1.00000 \\ \cos 84^\circ = \ .10453\end{array}\right\}\text{ subtract, since } \cos 96^\circ \text{ is } \textit{negative.}$$

$2)\ .89547$

$.44773$

Diff. $= .03315$

Product $= .01484, 22495 = \sin 51'$

$\therefore \tan 48^\circ\ 51' = 1.14376$

(4.) Given $\tan = 1.63422$

$\tan 58^\circ = 1.60033$

Diff. $= .03389$

$2 \times 58^\circ = 116^\circ$

$\cos 116^\circ = -\cos 64^\circ$

$$\begin{array}{rl} & 1.00000 \\ \cos 64° = & .43837 \end{array}\Big\}\text{ subtract}$$

$$\begin{array}{rl} 2) & \underline{.56163} \\ & .28081 \\ \text{Diff.} = & \underline{.03389} \end{array}$$

Product $=.00951, 66509 = \sin 33'$ more nearly than $\sin 32'$

$$\therefore \tan 58° 33' = 1.63422$$

if we use the Tables and Rules of the Manual.

SOLUTIONS

TO

THE NEW QUESTIONS IN THE THIRD EDITION OF THE MANUAL OF MECHANICS.

Page 12.

(1.) By equations (1) and (2) we have

$$Ph = wnt$$

$$\therefore n = \frac{Ph}{wt} = \frac{750 \times 2240 \times 6 \times 100}{33000 \times 60 \times 12} = \frac{75 \times 224}{33 \times 12} = \frac{25 \times 56}{11 \times 3}$$

$$= 42.\dot{4}\dot{2} \text{ H. P.} \quad \textit{Ans.}$$

(2.) Here we have, as in the last example,

$$Ph = wnt$$

$$\therefore t = \frac{Ph}{wn} = \frac{5000 \times 80 \times 10 \times 60}{33000 \times 20} = \frac{5 \times 4 \times 10 \times 20}{11}$$

$$= 363\tfrac{7}{11} \text{ minutes} = 6^{\text{h}}\ 3\tfrac{7}{11}{}^{\text{m}}. \quad \textit{Ans.}$$

(3). Since $Ph = wnt$, we have

$$w = \frac{Ph}{nt} = \frac{154 \times 5280 \times 2}{1 \times 60} = 154 \times 88 \times 2$$

$$= 27104 \text{ ft. lbs.} \quad \text{Ans.}$$

Page 13.

(4.) $$Ph = wnt$$

$$\therefore P = \frac{wnt}{h} = \frac{33000 \times 1310 \times 60}{73 \times 6} = \frac{33000 \times 1310 \times 10}{73}$$

$= 5921917.8$ lbs. (see remark following Equation (1) p. 10) $= 592191.78$ gallons, since the imperial gallon of water weighs 10 lbs. avoirdupois.

(5.) $$Ph = wnt$$

$$\therefore h = \frac{wnt}{P} = \frac{17536 \times 4 \times 60}{32130} = \frac{17536 \times 4 \times 2}{1071} = \frac{140288}{1071}$$

$$= 130.99 \text{ ft.} \quad \textit{Ans.}$$

(6.) $$Ph = wnt$$

$$\therefore t = \frac{Ph}{wn} = \frac{10000 \times 10 \times 40}{3897 \times \frac{2}{3} \times 20} = \frac{10000 \times 10 \times 2}{1299 \times 2} = \frac{100000}{1299}$$

$$= 77^{m} \textit{ very nearly} = 1^{h}\ 17^{m}. \quad \textit{Ans.}$$

(7.) $$P = \frac{wnt}{h} = \frac{467 \times 500 \times 6000}{5} = 467 \times 100 \times 6000 \text{ lbs.}$$

$$= \frac{467 \times 100 \times 6000}{2240 \times 2} \text{ cubic yards} = \frac{467 \times 100 \times 300}{224}$$

$$= \frac{14010000}{224} = 62544.6 \text{ cubic yards.} \quad \textit{Ans.}$$

Page 14.

(8.) $$P = \frac{wnt}{h} = \frac{1126 \times 30 \times 420}{30} = 1126 \times 420 \text{ lbs.}$$

$$= 1126 \times 420 \times \frac{17}{125} \text{ bricks} = \frac{1126 \times 420 \times 17 \times 8}{125 \times 8}$$

$$= \frac{1126 \times 420 \times 17 \times 8}{1000} = 11.26 \times 42 \times 17 \times 8$$

$$= 64317.12 \text{ bricks.} \quad \textit{Ans.}$$

(9.) $$wnt = w'\ n'\ t'$$

$$\therefore t' = \frac{wnt}{w'n'} = \frac{2598 \times 2 \times 10 \times 8 \times 60}{1126 \times 4} = \frac{2598 \times 2 \times 10 \times 60}{563}$$

$$= 5537.4^{\text{m}} = 92^{\text{h}}\ 17.4^{\text{m}} = 15^{\text{d}}\ 2^{\text{h}}\ 17.4^{\text{m}},$$

since the men work only 6 *hours daily.*

(10.) $$n' = \frac{wnt}{w'\,t'} = \frac{15588 \times 8 \times 24 \times 60}{33000 \times 24 \times 60} = \frac{5.196 \times 8}{11}$$

$$= \frac{41.568}{11} = 3.779 \text{ H. P. } \textit{very nearly.} \quad \textit{Ans.}$$

Page 82.

(12.) $$r = 17250 \text{ miles} = 91080000 \text{ ft.}$$

$$T = 9^{\text{h}}\ 30^{\text{m}} = 34200 \text{ seconds,}$$

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 91080000}{34200^2}$$

$$= \frac{9.8695877 \times 9108}{171^2} = \frac{9.8695877 \times 1012}{57^2}$$

$$= \frac{9988.0227524}{3249} = 3.0741836 \text{ ft. } \textit{Ans.}$$

(13.) Here (the radius of the moon's orbit)

$$r = 59.96435 \times \frac{7925.6}{2} = 59.96435 \times 3962.8 \text{ miles.}$$

$$= 59.96435 \times 3962.8 \times 5280 \text{ ft.} = 1254669114.2304 \text{ ft.}$$

$$\text{and } T = 27^d\ 7^h\ 43^m\ 11^s = 2360591 \text{ seconds;}$$

$$\therefore f = \frac{4\pi^2 r}{T^2} = \frac{4 \times 9.8695877 \times 1254669114.2304}{2360591^2}$$

$$= \frac{49532267429.51300322432}{5572389869281} = .00888887 \text{ ft. } \textit{Ans.}$$

(14.) By the last exercise the moon's distance from the earth $= 59.96435$ radii of the earth; $\therefore$ if x represent gravity at the distance of the moon, we have,

$$59.96435^2 : 1^2 :: 32.2 : x$$

$$\therefore x = \frac{32.2}{59.96435^2} = \frac{32.2}{3595.7232709225}$$

$$= .00895508 \text{ ft. } \textit{Ans.}$$

Page 85.

(1.) Here, $l = 72$, $a = 50$, $x = 30$, and $g = 32$,

$\therefore$ by equation (18) we have

$$v^2 = \frac{g}{l}(a^2 - x^2) = \frac{32}{72}(50^2 - 30^2)$$

$$= \frac{4}{9}(2500 - 900) = \frac{6400}{9},$$

$$\therefore v = \sqrt{\left(\frac{6400}{9}\right)} = \frac{80}{3} = 26\tfrac{2}{3} \text{ ft.} \quad \textit{Ans.}$$

(2.) Suppose AB (fig. p. 84) to be the horizontal diameter, and X the middle point of the arc AB, then

$$a^2 = AB^2 = 2l^2 = 2 \times 100^2 = 20000.$$

Since BX subtends at the centre G an angle $= 45°$, $\therefore$ the angle $BDX = 22° \ 30'$; and as the angle BXD in a semicircle is right, $BX = BD \sin BDX$, that is

$$x = BX = 2l \sin 22° \ 30'$$
$$= 200 \sin 22° \ 30' = 200 \times .3826834$$
$$= 76.53668 \text{ and } x^2 = 5857.8633854224$$

$$\therefore v^2 = \frac{g}{l}(a^2 - x^2) = \frac{32.1908}{100}(20000 - 5857.86)$$

$$= .321908 \times 14142.14 = 4552.46800312$$

$$\therefore v = 67.47 \text{ ft.} \quad \textit{Ans.}$$

Page 89.

(1.) By equation (21) we have

$$L = \frac{g}{\pi^2} = \frac{32.182}{3.14159^2} \text{ft.} = \frac{32.182 \times 12}{9.8695877}$$

$$= 39.128 \text{ inches. } \textit{Ans.}$$

(2.) By equation (23) we have

$$g = \pi^2 L = 9.8695877 \times 39.139 \text{ inches.}$$
$$= 386.2857929903 \text{ inches.}$$
$$= 32.1905 \text{ feet, } \textit{very nearly}. \quad \textit{Ans.}$$

(3.) Here, by equation (21) and the Table, p. 64, we have

$$L = \frac{g}{\pi^2} = \frac{32.2526}{9.8695877} \text{ ft.} = \frac{32.2526 \times 12}{9.8695877} \text{ inches.}$$

$$= \frac{387.0312}{9.8695877} = 39.2144 \text{ inches. } \textit{Ans.}$$

(4.) Here, by equation (21), and Table, p. 64, we have,

$$L = \frac{g}{\pi^2} = \frac{32.0881 \times 12}{9.8695877} \text{ inches.}$$

$$= \frac{385.0572}{9.8695877} = 39.0144 \text{ inches. } \textit{Ans.}$$

(5.) By example (2) $L = 39.139$ inches; also $T = 2^s$, $\therefore$ by equation (22) we have,

$$= LT^2 = 39.139 \times 2^2 = 156.556 \text{ inches}$$

$$= 13.046 \text{ ft. } \textit{Ans.}$$

(6.) Here $L = 39.139$ inches, and $T = \frac{1}{2}$ second;

$\therefore l = LT^2 = 39.139 \times \frac{1}{4} = 9.784$ inches. *Ans.*

Page 90.

(1.) Here $2\lambda = 2 \times 53^\circ\ 20' = 106^\circ\ 40'$

$\therefore \cos 2\lambda = -\cos(180^\circ - 106^\circ\ 40') = -\cos 73^\circ\ 20'$;

hence, by equation (24) we have,

$$L = 39.118 + \tfrac{1}{10} \cos 73^\circ\ 20'$$
$$= 39.118 + \tfrac{1}{10} \times .2868032 = 39.118 + .02868032$$
$$= 39.14668032 \text{ inches.}$$

By equation (23) we have,

$$g = \pi^2 L = 9.8695877 \times 39.146 \text{ inches.}$$
$$= 386.3548801042 \text{ inches,}$$
$$= 32.196 \text{ feet.}$$

(2.) $2\lambda = 2 \times 55^\circ\ 57' = 111^\circ\ 54'$

$\therefore \cos 2\lambda = -\cos(180^\circ - 111^\circ\ 54') = -\cos 68^\circ\ 6'$

$\therefore$ by equation (24),

$$L = 39.118 + \tfrac{1}{10} \cos 68^\circ\ 6'$$
$$= 39.118 + .03729878 = 39.15529878 \text{ inches;}$$

and $g = \pi^2 L = 9.8695877 \times 39.155$

$$= 386.4437063935 \text{ inches}$$
$$= 32.2036 \text{ ft.}$$

(3.) At the Equator $\lambda = 0$ $\therefore \cos 2\lambda = 1$

and $\therefore L = 39.118 - \frac{1}{10} = 39.018$ inches;

also, $g = \pi^2 L = 9.8695877 \times 39.018$

$= 385.0915728786$ inches $= 32.091$ ft.

Again, at the Pole $\lambda = 90^\circ$,

$$\therefore \cos 2\lambda = \cos 180^\circ = -1$$

and $L = 39.118 + \frac{1}{10} = 39.218$ inches;

$$g = \pi^2 L = 9.8695877 \times (39.018 + .2)$$

(by the last question) $= 385.0915728786 + 1.97391754$
$= 387.06549041186$ inches $= 32.255$ ft.

Page 91.

(1.) Here $n = 86400^s$, the number of seconds in a mean solar day;

$$g = 32.1908 \text{ (Table, p. 64)}$$

$g' = 32.0881$ (Table, p. 64, the Isle of Rawak being on the Equator).

Hence, by equation (27) we have,

$$n - n' = \frac{n}{2} \times \frac{g - g'}{g} = 43200 \times \frac{32.1908 - 32.0881}{32.1908}$$

$$= 43200 \times \frac{1027}{321908} = \frac{44366400}{321908} = 137.8^s \quad \textit{Ans.}$$

(2.) Here $\qquad n = 85945.8$

$$n' = 85933.83$$

and we are to calculate the value of the fraction $\dfrac{g - g'}{g}$:

by equation (27) we have,

$$n - n' = \frac{n}{2} \times \frac{g - g'}{g} \therefore \frac{g - g'}{g} = \frac{2(n - n')}{n}$$

$$=\frac{2\,(85945.8-85933.83)}{85945.8}$$

$$=\frac{2\times 11.97}{85945.8}=\frac{23.94}{85945.8}=\frac{1}{3590}\quad \textit{Ans.}$$

(3.) Here $n = 86400^s$, and $n' - n = 2\frac{1}{4}^s$;

we are required to calculate $\frac{g'-g}{g}$, $\therefore$ by equation (27) we have,

$$\frac{g'-g}{g}=\frac{2\,(n'-n)}{n}=\frac{2\times 2\frac{1}{4}}{86400}$$

$$=\frac{9}{172800}=\frac{1}{19200}\quad \textit{Ans.}$$

Page 93.

(1.) Here $l' = 45$ inches, and $l = 45 - \frac{1}{32}$ inches.

$$\therefore \frac{l'-l}{l'}=\frac{1}{32}\div 45=\frac{1}{32\times 45}=\frac{1}{1440};$$

hence by equation (29), we have,

$$\text{Acceleration in a day} = 43200\times\frac{l'-l}{l'}$$

$$=43200\times\frac{1}{1440}=30 \text{ seconds.}\quad \textit{Ans.}$$

(2.) Here $T' - T = 2^m = 120^s$, and $l = 39.14$ inches (the length of the seconds pendulum in London, see p. 88), $\therefore$ by equation (29) we have,

$$120 = 43200\times\frac{l'-l}{l},\ \therefore\ 1 = 360\times\frac{l'-l}{l}$$

$$\text{and } l' - l = \frac{l}{360} = \frac{39.14}{360} = \frac{1957}{18000} \text{ inches;}$$

the number of turns (x) of the screw, to which this is equivalent is found by multiplying it by 50,

$$\therefore x = \frac{1957 \times 50}{18000} = \frac{1957}{360} = 5.4 \quad \textit{Ans.}$$

Page 94.

(3.) Here (see last example) L, the length of the seconds pendulum in London, = 39.14 inches,

$$n = \text{the seconds in } 24^{d}\ 3^{m}\ 56.5^{s} = 86636.5^{s}$$

and we are required to find l from the equation,

$$n^2 = \frac{L}{l} \times (86400)^2;$$

$$\therefore l = L \times \left(\frac{86400}{86636.5}\right)^2$$

$$= 39.14 \times \left(\frac{172800}{173273}\right)^2$$

$$= 39.14 \times \left(1 - \frac{473}{173273}\right)^2 = 39.14 \times \left(1 - \frac{946}{173273}\right) \textit{very nearly},$$

$$= 39.14 - .213 = 38.927 \text{ inches.} \quad \textit{Ans.}$$

(4.) By the Table, p. 93, the value of

$\frac{l'-l}{l}$ *for* 10° *F. in the* case of a brass pendulum is $\frac{1}{9600}$

$$\therefore \text{ for } 1^\circ \text{ F. its value is } \frac{1}{10} \times \frac{1}{9600} = \frac{1}{96000};$$

hence by equation (29) we have,

$$\text{Retardation} = 43200 \times \frac{1}{96000} = \frac{43.2}{9}$$

$$= .45 \text{ seconds. } \textit{Ans.}$$

5.) By Table, p. 93,

$$\frac{l'-l}{l} = \frac{1}{14400} \text{ for } 10^\circ \text{ F}$$

$\therefore$ by equation (29),

$$\text{Acceleration} = 43200 \times \frac{1}{14400} = 3 \text{ seconds. } \textit{Ans.}$$

(6.) Here by Table, p. 93,

$$\frac{l'-l}{l} = \frac{13^\circ}{10^\circ} \times \frac{1}{41000} = \frac{13}{410000}$$

$\therefore$ by equation (29),

$$\text{Retardation} = 43200 \times \frac{13}{410000} = \frac{56.16}{41}$$

$$= 1.37 \text{ seconds. } \textit{Ans.}$$

(7.) Here by Table, p. 93,

$$\frac{l'-l}{l} = \frac{14^\circ}{10^\circ} \times \frac{1}{6120} = \frac{7}{30600}$$

$\therefore$ by equation (29) we have,

$$\text{Acceleration} = 43200 \times \frac{7}{30600} = \frac{3024}{306}$$

$$= 9.88 \text{ seconds. } \textit{Ans.}$$

(8.) Here *increase* of temperature

$$= 59.5^\circ - 42^\circ = 17^\circ.5$$

$\therefore$ by Table p. 93, we have,

$$\frac{l'-l}{l} = \frac{17.5^\circ}{10^\circ} \times \frac{1}{15600} = \frac{7}{62400}$$

$\therefore$ by equation (29) we have,

$$\text{loss in 1 day} = 43200 \times \frac{7}{62400} = \frac{36 \times 7}{52} = \frac{63}{13}$$

$$\text{and } \therefore \text{ loss in a week} = \frac{63}{13} \times 7 = \frac{441}{13}$$

$$= 33.92 \text{ seconds.} \quad \textit{Ans.}$$

(9.) Here we shall use equation (22), viz., $l = LT^2$, where l is the required length, L the length of a seconds pendulum in London (39.14 inches, see p. 88), and

$$T = \frac{3 \times 60}{48} = 3.75^s$$

$\therefore l = 39.14 \times 3.75^2 = 39.14 \times 14.0625 = 550.40625$ inches.

$$= 45.87 \text{ ft.} \quad \textit{Ans.}$$

(10.) Here we shall employ equation (20), viz.,

$$T = \pi \sqrt{\left(\frac{l}{g}\right)},$$

where T is the duration of one vibration in seconds,

$g = 32.1908$ ft., and $l = 340$ ft. $-$ 6 in. $= 339.5$ ft.

$$\therefore T = 3.1416 \sqrt{\frac{339.5}{32.1908}} = 3.1416 \sqrt{(10.5465)}$$

$$= 3.1416 \times 3.247 = 10.2007752 \text{ seconds;}$$

$\therefore$ the number of vibrations in half an hour equals

$$\frac{30 \times 60^s}{10.2^s} = \frac{1800}{10.2} = 176.4 \quad \text{Ans.}$$

SOLUTIONS

OF

THE NEW QUESTIONS IN THE FOURTH EDITION OF THE MANUAL OF MECHANICS.

Pages 15, 16.

(1.) In last figure, page 4, let

OA = P, OB = Q, OC = R, and the angle AOB = ϕ; then, by Trigonometry,

$OC^2 = OA^2 + CA^2 - 2OA \cdot CA \cos OAC$, but cos OAC = − cos AOB, since these ∠ˢ are supplemental; also,

$$CA = OB = Q,$$

$$\therefore R^2 = P^2 + Q^2 + 2PQ \cos \phi.$$

(2.) Using the same figure, let OA = P, OB = Q, then OC is equal and *opposite* to *R*.

Now from Δ AOC we have

$AO : OC : CA = \sin OCA : \sin CAO : \sin AOC$, and

$$\sin OCA = \sin COB = \sin BOC' = \sin \widehat{QR};$$

$$\sin CAO = \sin AOB = \sin \widehat{PQ};$$

$$\sin AOC = \sin AOC' = \sin \widehat{RP};$$

∴ by substitution,

$$P : R : Q = \sin \widehat{QR} : \sin \widehat{PQ} : \sin \widehat{RP}.$$

N. B.—Since the sides of the Δ OCA are parallel to the three forces which are in equilibrium, it is plain that when any three forces are in equilibrium the sides of any Δ parallel to them, taken in *the same order*, are proportional to the forces.

(3.) See (16), p. 8, "Key."

(4.) See (17), p. 9, "Key."

(5.) See (19), p. 9, "Key."

(6.) See (21), p. 10, "Key."

(7.) See (22), p. 10, "Key."

(8.) Fig., p. 4. Let OC = 10 lbs., OA = 8 lbs., then $\angle$ ACO = 36°, and AC and $\angle$ AOB are required.

$$\frac{OC}{OA} = \frac{\sin OAC}{\sin ACO} = \frac{\sin AOB}{\sin ACO}$$

$$\therefore \frac{10}{8} = \frac{\sin AOB}{\sin 36^\circ}$$

$$\therefore \sin AOB = \tfrac{5}{4} \sin 36^\circ$$

$$= \tfrac{5}{4} \times .5877853 = .7347316$$

$$= \sin 47^\circ\ 17'; \ AOC = 47^\circ\ 17' - 36^\circ = 11^\circ\ 17'.$$

Again,
$$AC^2 = AO^2 + OC^2 - 2AO.OC \cos AOC$$

$$= 64 + 100 - 2 \times 8 \times 10 \cos 11^\circ\ 17'$$

$$= 64 + 100 - 2 \times 8 \times 10 \times .9806716$$

$$= 164 - 156.907456 = 7.092544$$

$$\therefore AC = \sqrt{(7.092544)} = 2.663.$$

(9.) Let ABC be a Δ; P, Q, R, the middle points of AB, BC, CA; and O the centre of the circumscribed circle. Since the forces P, Q, R act perpendicularly to the sides through their middle points, their directions meet in O. Now by the hypothesis we have

$$P : Q : R = AB : BC : AC$$
$$= \sin C : \sin B : \sin A$$
$$= \sin \widehat{RQ} : \sin \widehat{QP} : \sin \widehat{PR};$$
$$\text{since } C + \widehat{RQ} = 180°, \text{ \&c.}$$

$\therefore$ the forces are in equilibrium by Ex. (2).

(10.) Since the sides of the Δ COA are proportional to P, Q, R, $\therefore$ its $\angle^s$ can be found; hence if QP be joined, in the Δ POQ we have $\angle$ POQ and $\angle$ QPO $= \angle$ QPA′ − POC, both of which are known, $\therefore$ the $\angle^s$ of the Δ POQ and its side PQ being known, the position of equilibrium is determined.

(11.) The resolved parts of P and Q along their planes are P $\sin\alpha$, Q $\sin\beta$, but these are the tensions on the string,

$$\therefore P \sin\alpha = Q \sin\beta,$$
$$\therefore P : Q :: \sin\beta : \sin\alpha.$$

(12.) We know from Trigonometry that

$$\begin{cases} \sin 105° = \cos 15° = \frac{1}{4}(\sqrt{6} + \sqrt{2}) \\ \cos 105° = -\sin 15° = -\frac{1}{4}(\sqrt{6} - \sqrt{2}) \end{cases}$$

$$\begin{cases} \sin 120° = \sin 60° = \frac{1}{2}\sqrt{3} \\ \cos 120° = -\cos 60° = -\frac{1}{2} \end{cases}$$

$$\begin{cases} \sin 135° = \sin 45° = \frac{1}{2}\sqrt{2} \\ \cos 135° = -\cos 45° = -\frac{1}{2}\sqrt{2} \end{cases}$$

Let
$$P = \sqrt{3} + 1$$
$$Q = \sqrt{6}$$
$$R = 2$$

Now since the three forces are in equilibrium, each force is equal and opposite to the resultant of the other two; hence, by Ex. (1), we have

$$(1) \quad R^2 = P^2 + Q^2 + 2PQ \cos \widehat{PQ}$$

$$(2) \quad Q^2 = P^2 + R^2 + 2PR \cos \widehat{PR}$$

$$(3) \quad P^2 = Q^2 + R^2 + 2QR \cos \widehat{QR}$$

(1) becomes, by substitution,

$$4 = 4 + 2\sqrt{3} + 6 + 2\sqrt{6}(\sqrt{3} + 1) \cos \widehat{PQ}$$

$$\therefore \cos \widehat{PQ} = -\frac{\sqrt{3} + 3}{\sqrt{6}(\sqrt{3} + 1)} = -\frac{1}{\sqrt{2}}$$

$$= -\tfrac{1}{2}\sqrt{2} = \cos 135^\circ$$

(2) becomes

$$6 = 4 + 2\sqrt{3} + 4 + 4(\sqrt{3} + 1) \cos \widehat{PR}$$

$$\therefore \cos \widehat{PR} = -\frac{2 + 2\sqrt{3}}{4(\sqrt{3} + 1)} = -\tfrac{1}{2} = \cos 120^\circ;$$

$$\text{hence } \widehat{QR} = 360^\circ - 105^\circ - 120^\circ = 135^\circ.$$

(13.) Let R be the resultant of all the forces, and a its inclination to the first force; also, let R_1 be resultant of forces resolved along first force, and R_2 along third force.

$$\therefore R_1 = 1 + 2 \cos 60^\circ + 4 \cos (90^\circ + 60^\circ)$$
$$= 1 + 1 - 4 \cos 30^\circ = 2 - 2\sqrt{3}$$

which being negative, acts in a direction opposite to 1.

Again, resolve along the third force,

$$\therefore R_2 = 3 + 2\cos(90^\circ - 60^\circ)$$
$$+ 4\cos 60^\circ = 3 + \sqrt{3} + 2 = 5 + \sqrt{3}$$
$$\therefore R = \sqrt{\{(5+\sqrt{3})^2 + (2 - 2\sqrt{3})^2\}}$$
$$= \sqrt{(44 + 2\sqrt{3})} = \sqrt{(44 + 3.464)} = 6.9 \text{ lbs.};$$

also $$\cos\alpha = \frac{2 - 2\sqrt{3}}{6.9} = \frac{2 - 3.464}{6.9}$$
$$= -\frac{1.464}{6.9} = -.212174$$
$$= \cos(180^\circ - 77^\circ 45') = \cos 102^\circ 15'.$$

(14.) OA = 27, OB = 52, OC = 49, taken in the order of the letters A, B, C.

$$\therefore \angle AOB = 32^\circ,\ BOC = 26^\circ \text{ and } AOC = 58^\circ.$$

Let R be required resultant, α the required $\angle$.

R_1 = sum of resolved parts in direction of OC,
R_2 = sum of resolved parts $\perp^r$ to OC.

$$\therefore R_1 = 49 + 52\cos 26^\circ + 27\cos 58^\circ$$
$$= 49 + 52 \times .8987940$$
$$+ 27 \times .5299193$$
$$= 49 + 46.737288 + 14.3078211$$
$$= 110.0451091$$

$$R_2 = 52\sin 26^\circ + 27\sin 58^\circ$$
$$= 52 \times .4383711 + 27 \times .8480481$$
$$= 22.7952972 + 22.8972987$$
$$= 45.6925959$$

$$\therefore R = \sqrt{\{(110.05)^2 + (45.69)^2\}}$$
$$= \sqrt{(12111.0025 + 2087.5761)}$$
$$= 119.16 \text{ required resultant,}$$
$$\therefore \cos\alpha = \frac{R_1}{R} = \frac{110}{119.2} = .92366$$
$$= \cos 22^\circ 32'.$$

(15.) Using the same notation as in the last Example, we have

$$\angle AOC = 17^\circ + 52^\circ = 69^\circ.$$

$$R_1 = 29 + 16 \cos 52^\circ + 31 \cos 69^\circ$$
$$= 29 + 16. \times .61566 + 31 \times .35837$$
$$= 49.96.$$

$$R_2 = 16 \sin 52^\circ + 31 \sin 69^\circ$$
$$= 16 \times .788 + 31 \times .93358$$
$$= 41.54.$$

$$\therefore R = \sqrt{\{(49.96)^2 + (41.54)^2\}}$$
$$= \sqrt{(2496.0016 + 1725.5716)}$$
$$= 64.98 \text{ lbs.}$$
$$\therefore \cos\alpha = \frac{R_1}{R} = \frac{49.96}{64.98} = .76885$$
$$= \cos 39^\circ 45'$$

(16.) The ring will plainly rest at C, the middle point of the cord $2l$, and if the vertical through C meet the line AB in D, then AD = DB = a, and

$$\angle ACD = BCD = \phi$$
$$\therefore \sin\phi = \frac{a}{l} \text{ and } \cos\phi = \frac{\sqrt{(l^2 - a^2)}}{l}.$$

Now the sum of the tensions of CA, CB, resolved vertically, = W, that is,

$$2T \cos \phi = W$$

$$\therefore T = \frac{W}{2 \cos \phi} = \frac{Wl}{2\sqrt{(l^2 - a^2)}}$$

(17.) Fig., p. 102.

Suppose G the centre of the wheel, which is also its centre of gravity, through which the weight W and the force F act; the former vertically, the latter horizontally. Through X draw a $\perp$' to the tangent at B, meeting it in A, and join XG. Let XA be the obstacle $= h$. The resistance R of the obstacle must pass through G, since there is equilibrium, and W, F, G being parallel to the sides GH, HX, XG, of the Δ GHX, taken in order, must be proportional to them,

$$\therefore W : F :: GH : HX,$$

but $HG = r - h$, and

$$HX = \sqrt{(BH \,.\, HD)} = \sqrt{\{h(2r - h)\}}$$

$$\therefore W : F :: r - h : \sqrt{(2rh - h^2)}$$

$$F = W \,.\, \frac{\sqrt{(2rh - h^2)}}{r - h}. \quad \textit{Ans.}$$

(18.) Since the forces P, Q, R equilibrate at a point, they are proportional to the sides of a Δ, drawn parallel to them, and in the same directions; but if a, b, c be the sides, and A, B, C the $\angle$ˢ respectively opposite to them, we have, by a common trigonometrical formula,

$$\sin A = \frac{\sqrt{\{(a+b+c)(a+b-c)(b+c-a)(c+a-b)\}}}{2bc}$$

Now A is $\angle$ included by b, c, and $\widehat{PQ}$ is $\angle$ included by P and Q,

$$\therefore \sin \widehat{PQ} = \frac{\sqrt{\{(P+Q+R)(Q+R-P)(R+P-Q)(P+Q-R)\}}}{2PQ}$$

Pages 50, 51, 52.

(1.) Let a, b, and c denote the tensions of AC, BD, and CD respectively.

The point C is kept in equilibrium by the three forces a, P, c acting in the directions of CA, Cc, and CD, and the sides of the Δ CDd are parallel to these directions, and $\therefore$ proportional to the forces: hence

$$a : P : c = Cd : Dd : CD. \qquad (1)$$

Similarly D is kept in equilibrium by b, Q, and c acting parallel to sides of Δ CDc.

$$\therefore b : Q : c = Dc : Cc : CD. \qquad (2)$$

From (1) we have

$$\frac{P}{c} = \frac{Dd}{CD}.$$

From (2),

$$\frac{c}{Q} = \frac{Cd}{Cc}$$

$$\therefore \frac{P}{c} \cdot \frac{c}{Q} = \frac{Dd}{CD} \cdot \frac{CD}{Cc},$$

that is,

$$\frac{P}{Q} = \frac{Dd}{Cc}. \quad \textit{Ans.}$$

(2.) From equations (1) and (2) of last question, we have

$$\frac{a}{P} = \frac{Cd}{Dd} \text{ and } \frac{Q}{b} = \frac{Cc}{Dc}$$

$$\therefore \frac{a}{P} \cdot \frac{Q}{b} = \frac{Cd}{Dd} \cdot \frac{Cc}{Dc}$$

that is,

$$\frac{a}{b} \cdot \frac{Q}{P} = \frac{Cc}{Dd} \cdot \frac{Cd}{Dc}.$$

Now, if $\frac{Q}{P} = \frac{Cc}{Dd}$, then

$$\frac{a}{b} = \frac{Cd}{Dc}. \qquad Q. E. D.$$

(4.) Plate III., Fig. 1.

Let AD, BD be the inclined planes, DF horizontal, A and B the points of contact of the sphere whose centre is C, with the planes, R, R′, the normal pressures of the planes on the sphere at A and B respectively. R and R′ must act through C; hence if the vertical AG meet BC in G, and the horizontal line in F, R, R′ and W will be proportional to the sides of the $\triangle$ ACG, since the sphere is kept in equilibrium by R, R′, W.

$$\therefore\ R : R' : W = AC : CG : GA$$
$$= \sin G : \sin GAC : \sin ACG$$
$$= \sin i' : \sin i : \sin (i + i'), \text{ for}$$

$\angle$ ACG = ADB (since quadrilateral ADBC has $\angle^s$ at A and B right) $= 180^\circ - (i + i')$;

also $\angle$ GAC = ADF, for they have a common complement DAF, and $\angle G = 180^\circ - (GAC + ACG)$,

$$= 180^\circ - \{i + 180^\circ - (i + i')\}$$
$$= i'.$$

hence $R : R' : W = \sin i' : \sin i : \sin (i + i')$ gives

$$R = \frac{W \sin i'}{\sin (i + i')}$$

$$R' = \frac{W \sin i}{\sin (i + i')}.$$

(5.) Plate III., Fig. 2.

Let BC be the vertical wall, AC horizontal, and A the fixed peg against which the ladder AB presses, G centre of gravity of ladder, and F the point in which vertical through G meets BF $\perp^r$ to the wall.

Since the wall is smooth, it can only exert a normal pressure P on the ladder, and since two of the three forces, viz. P and W, which keep the ladder in equilibrium, meet in F, the direction of the third, Q, must also pass through F.

Since the Δ AFD has its sides parallel to the directions of P, Q, W, we have

$$P : Q : W = AD : AF : FD.$$

Now $AC = BC \tan\theta = FD \tan\theta$, and $AD : DC = m : n$

$$\therefore AD = \frac{m}{m+n} \cdot AC = \frac{m \tan\theta}{m+n} \cdot FD$$

$$AF = \sqrt{(AD^2 + FD^2)} = FD\sqrt{\left(1 + \frac{m^2 \tan^2\theta}{(m+n)^2}\right)}$$

$$\therefore P : Q : W = \frac{m \tan\theta}{m+n} \cdot FD$$

$$: FD\sqrt{\left\{1 + \left(\frac{m^2 \tan^2\theta}{(m+n)^2}\right)\right\}} : FD,$$

that is, $$P : W = \frac{m \tan\theta}{m+n} : 1$$

and $$Q : W = \sqrt{\left(1 + \frac{m^2 \tan^2\theta}{(m+n)^2}\right)} : 1$$

$$\therefore P = W \cdot \frac{m}{m+n} \cdot \tan\theta$$

$$Q = W \cdot \sqrt{\left(1 + \frac{m^2 \tan^2\theta}{(m+n)^2}\right)}.$$

(6.) Plate III., Fig. 2.

Suppose B the hinge, F the pulley, and BL the position of the door in equilibrium, the $\angle$ FBA being *here* $= \theta$. The moments of P and W about B must be equal.

Now BM $\perp^r$ to FL is $= l \cos \frac{1}{2}\theta$, since FB = BL = l, and $\perp^r$ from B on direction of W, which acts vertically through middle point of BL, is $= \frac{1}{2}l \cos \theta$.

$$\therefore P \cdot l \cos \tfrac{1}{2}\theta = W \cdot \tfrac{1}{2}l \cos \theta,$$

$$\text{and } \therefore P = W \cdot \frac{\cos \theta}{2 \cos \frac{1}{2}\theta}. \quad \textit{Ans.}$$

(7.) Plate III., Fig. 3.

Let the beam AB rest on the two planes AM, BM, and CG be the vertical through G, its centre of gravity ($\therefore$ AG = GB). AC is normal to AM, and BC $\perp^r$ to BM, meets the vertical AF in H.

Since the Δ ACH has its sides parallel to the directions of P, P′, W, we have

$$P : P' : W = AC : CH : HA$$
$$= \sin CHA : \sin HAC : \sin ACH.$$

Now $\angle$ CAH = i, since they have a common complement MAF.

CHA = BCD = i', since i' and BCD have the common supplement, BMD and ACH = 180°

$$- (CHA + HAC) = 180^\circ - (i + i')$$
$$\therefore P : P' : W = \sin i' : \sin i : \sin (i + i')$$

and
$$\therefore P = \frac{W \sin i'}{\sin (i + i')}$$
$$P' = \frac{W \sin i}{\sin (i + i')}.$$

Again, in Δ BCA we have BCG = i', ACG = i, and BG = AG,

$$\therefore \frac{BG}{CG} = \frac{\sin i'}{\sin B}$$

$$\frac{AG}{CG} = \frac{\sin i}{\sin A}$$

$$\therefore \frac{\sin A}{\sin B} = \frac{\sin i}{\sin i'}$$

$$\therefore \frac{\sin (i + i' + B)}{\sin B} = \frac{\sin i}{\sin i'}$$

$$\therefore \sin (i + i') \cot B + \cos (i + i') = \frac{\sin i}{\sin i'}$$

that is, $\sin (i + i') \tan \phi = \dfrac{\sin i}{\sin i'} - \cos (i + i')$. (1)

But $\tan \theta = \tan (\phi - i')$

$$= \frac{\tan \phi \sin (i + i') - \tan i' \sin (i + i')}{\sin (i + i') + \tan \phi \sin (i + i') \tan i'}$$

$\therefore$ substituting from (1),

$$\tan \theta = \frac{\dfrac{\sin i}{\sin i'} - \cos (i + i') - \tan i' \sin (i + i')}{\sin (i + i') + \dfrac{\sin i}{\sin i'} - \cos (i + i') \tan i'}$$

$$= \frac{\dfrac{\sin i}{\sin i'} - \dfrac{1}{\cos i'}\{\cos (i + i') \cos i + \sin i' \sin (i + i')\}}{\dfrac{1}{\cos i'}\{\sin (i + i') \cos i' - \cos (i + i') \sin i'\} + \dfrac{\sin i}{\sin i'}}$$

$$= \frac{\dfrac{\sin i}{\sin i'} - \dfrac{\cos i}{\cos i'}}{\dfrac{\sin i}{\cos i'} + \dfrac{\sin i}{\cos i'}} = \frac{\sin (i - i')}{2 \sin i \sin i'}.$$

(8.) Plate III., Fig. 2.

Let A be the point where half the roof AB rests on the side wall, and BC the vertical through its summit, then weight of AB = $\frac{1}{2}$W, and acts through G, middle point of AB, and $\therefore m = n$. We may suppose BC a fixed plane, since there is equilibrium; and if AD represent the normal reaction of BC on AB, then AF will represent the reaction of the point A in magnitude and direction, but AF resolved in horizontal direction,

$$AC \text{ is} = AD = P;$$

and P in question (5)

$$= W \cdot \frac{m}{m+n} \tan\theta,$$

but here weight of $\frac{1}{2}$ roof = $\frac{1}{2}$W, and $\phi = 90° - \theta$, also $m = n$; $\therefore$ Horizontal Thrust = $\frac{1}{4}$W $\cot\phi$.

Pages 88, 89, 90, 91.

(6.) Here sine of inclination of plane $= \frac{h}{l}$, $\therefore$ the part of W which acts down the plane is $W \cdot \frac{h}{l}$, and $\therefore f$, the acceleration *up* the plane, is

$$f = \frac{T - W \cdot \frac{h}{l}}{T + W} \cdot g;$$

but equation (4) gives

$$S = \tfrac{1}{2} f t^2, \text{ and here } S = l,$$

$$\therefore t^2 = \frac{2l}{f} = \frac{2l}{g} \cdot \frac{T + W}{T - W \cdot \frac{h}{l}}$$

$$= l^2 \cdot \frac{2}{g} \cdot \frac{T + W}{Tl - Wh}$$

$$\therefore t = l\sqrt{\left(\frac{2}{g} \cdot \frac{T + W}{Tl - Wh}\right)}. \quad \textit{Ans.}$$

(7.) Let t denote the time after the first stone is projected when the second must be projected, then if the ten seconds be measured from the starting of the first stone, equation (4 bis) gives for the two stones the two equations—

$$S = 50 \times 10 + 16 \times 100$$
$$S = 100(10 - t) + 16(10 - t)^2;$$

equating these two values of S, we have, after reduction,

$$16t^2 - 420t = -500.$$

The admissible value of t is

$$t = 1\tfrac{1}{4}S,$$

but if the 10 seconds be measured from starting of the second stone, the equations are,

$$S = 50(10 + t) + 16(10 + t)^2$$
$$S = 100 \times 10 + 16 \times 100.$$

Equating these values of S, we have

$$t = \frac{-185 \pm 205.5}{16}$$

The positive value of t is

$$t = 1.28^s.$$

(8.) Resolving the two weights along the planes, we have for the statical force which produces the acceleration f in the two weights—

$$W_1 \cdot \frac{h}{l_1} - W_2 \frac{h}{l_2}$$

$$\therefore f = \frac{W_1 \cdot \frac{h}{l_1} - W_2 \frac{h}{l_2}}{W_1 + W_2} \cdot g$$

$$= \frac{gh}{l_1 l_2} \cdot \frac{W_1 l_2 - W_2 l_1}{W_1 + W_2}. \quad \textit{Ans.}$$

(9.) The space described in the first $(t - 1)$ seconds is

$$s - \frac{s}{n} = \frac{1}{2} g (t - 1)^2,$$

and the entire space described in the t seconds is

$$s = \frac{1}{2} gt^2;$$

hence, substituting for s from this equation in the former, we have

$$\frac{1}{2} gt^2 \left(1 - \frac{1}{n}\right) = \frac{1}{2} g (t - 1)^2,$$

$$\therefore t^2 \left(1 - \frac{1}{n}\right) = (t - 1)^2,$$

$$\therefore t \sqrt{\left(1 - \frac{1}{n}\right)} = \pm (t - 1),$$

$$\therefore t \left\{ \sqrt{\left(1 - \frac{1}{n}\right)} \mp 1 \right\} = \mp 1,$$

$$t = \frac{\mp 1}{\sqrt{\left(1 - \frac{1}{n}\right)} \mp 1} = \frac{1}{1 \mp \sqrt{\left(1 - \frac{1}{n}\right)}},$$

$$= \frac{1 \pm \sqrt{1 - \frac{1}{n}}}{\frac{1}{n}} = n \pm \sqrt{\{n (n - 1)}. \quad \textit{Ans.}$$

(10.) The equation (4 bis), viz.,

$$S = Vt + \tfrac{1}{2} gt^2,$$

becomes

$$300 = 4V + 16 \times 4^2,$$

$$\therefore V = \frac{300 - 256}{4} = 11 \text{ ft.} \quad \textit{Ans.}$$

(11.) Let t_1, t_2 denote the times required by the first and second stones respectively to reach the middle point of the tower, then equation (4 *bis*) gives

$$150 = 100\, t_1 - 16\, t^2_1, \qquad (1)$$

$$150 = 100\, t_2 + 16\, t^2_2, \qquad (2)$$

From (1), $t_1 = 2\tfrac{1}{2}$ or $3\tfrac{3}{4}$.

From (2), $t_2 = 1\tfrac{1}{4}$ or $- 7\tfrac{1}{2}$;

$$\therefore t = t_1 - t_2 = 2\tfrac{1}{2} - 1\tfrac{1}{4} = 1\tfrac{1}{4}^s. \quad \textit{Ans.}$$

(12.) Let t_1, t_2 denote the times in which the first stone falls through 50 and 400 ft., and $t = t_2 - t_1$ time in which second stone falls through 400 ft.

$$\therefore 50 = 16\, t^2_1 \quad \therefore t_1 = \tfrac{5}{4}\sqrt{2},$$

$$400 = 16\, t^2_2 \quad \therefore t_2 = 5,$$

and

$$\therefore t = t_2 - t_1 = 5 - \tfrac{5}{4}\sqrt{2},$$

$$= \tfrac{5}{4}(4 - \sqrt{2});$$

hence by (4 bis) we have

$$400 = Vt + 16t^2,$$

$$\therefore V = \frac{400 - 16t^2}{t},$$

$$= \frac{400 - 25(18 - 8\sqrt{2})}{\frac{5}{4}(4 - \sqrt{2})},$$

$$= 40\,\frac{(4\sqrt{2} - 1)}{4 - \sqrt{2}}$$

$$= \frac{20}{7}(15\sqrt{2} + 4),$$

$$= \frac{20}{7}(15 \times 1.4142 + 4),$$

$$= 72.037 \text{ ft. } \quad \textit{Ans.}$$

(13.) Let i = inclination of plane, l = its length, and $\therefore l \sin i$ = its height, t = time of falling through $l \sin i$, and $\therefore nt$ = time of running down l.

$\therefore f = g \sin i$ = acceleration down inclined plane.

$\therefore$ by Equation (4), viz., $t^2 = \frac{2s}{f}$, we have

$$t^2 = \frac{2 \cdot l \sin i}{g} \text{ for falling through height.}$$

$$n^2 t^2 = \frac{2l}{g \sin i} \text{ for running down plane.}$$

$$\therefore \frac{2 \cdot l \sin i}{g} \times n^2 = \frac{2l}{g \sin i}$$

$$\therefore \sin^2 i = \frac{1}{n^2}, \text{ or } \sin i = \frac{1}{n}. \quad \textit{Ans.}$$

(14.) Let f = acceleration down plane, t = time of describing each of the n parts;

x_m = sum of first m parts.

∴ By equation (4) we have

$$x_m = \tfrac{1}{2} f(mt)^2$$

$$x_{m-1} = \tfrac{1}{2} f(m-1)^2 t^2$$

$$\therefore\ x_m - x_{m-1} = \tfrac{1}{2} ft^2 (2m-1);$$

$$\text{but } l = \tfrac{1}{2} f(nt)^2 \ \therefore\ \tfrac{1}{2} ft^2 = \frac{l}{n^2}$$

$$\text{and } \therefore\ x_m - x_{m-1} = \frac{l}{n^2}(2m-1) = \text{the } m^{th} \text{ part.}$$

The successive parts are now found by making m = 1, 2, 3, &c. up to n, and are

$$\therefore\ \frac{l}{n^2},\ 3\frac{l}{n^2},\ 5\frac{l}{n^2},\ \text{\&c.}\ (2n-1)\frac{l}{n^2}.$$

(15.) Here we have

$$l = Vt + \tfrac{1}{2} g \sin i \,.\, t^2$$

$$h = \tfrac{1}{2} gt^2$$

$$\therefore\ l = V\sqrt{\frac{2h}{g}} + h \sin i$$

$$\therefore\ V = \frac{l - h\sin i}{\sqrt{\frac{2h}{g}}} = \frac{g(l-h)\sin i)}{\sqrt{(2gh)}}. \quad \textit{Ans.}$$

(16.) Velocity acquired in falling through h is

$$v = \sqrt{(2gh)};$$

hence, substituting for V in the answer to the last, $\frac{3}{4}v$, we have

$$\tfrac{3}{4}\sqrt{(2gh)} = \frac{g(l - h\sin i)}{\sqrt{(2gh)}}$$

$$\therefore\ \tfrac{3}{2}h = l - h\sin i;$$

but $h = l\sin i$, and

$$\therefore\ \tfrac{3}{2}\sin i = 1 - \sin^2 i,$$

$$\text{or, } \sin i = \frac{-3 \pm 5}{4} = \tfrac{1}{2} \text{ or } -2,$$

-2 must, of course, be rejected;

$$\therefore\ \sin i = \tfrac{1}{2} = \sin 30^\circ. \quad \textit{Ans.}$$

(17.) Plate III., Fig. 4.

I shall first prove the following geometrical theorem:—

If $ABB''A''$ be a quadrilateral, and $AC : CB = A''C'' : C''B''$; $AA' : A'A'' = BB' : B'B''$, then the right lines $A'B'$, CC'' are also divided in C', so that

$$A'C' : C'B' = AC : CB,$$
$$CC' : C'C'' = AA' : A'A''\ ?$$

Draw CE parallel to AA'', CG parallel to BB'', and $A'D$, $A''E$, $B'F$, $B''G$ all parallel to AB. Also join

EG meeting $A''B''$ in x,
DF meeting CC'' in y.

Since $A''E = AC$ and $B''G = BC$, and the Δ^s $A''xE$, $B''xG$ are similar, $A''x : B''x = A''E : B''G = AC : BC$, but $A''C'' : B''C''$ is in same ratio, $\therefore$ x coincides with C'', and $EC'' : C''G = AC : CB$.

Again, $CD = AA'$, $DE = A'A''$, $CF = BB'$, $FG = B'B''$, and $AA' : A'A'' = BB' : B'B''$, $\therefore$ $CD : DE = CF : FG$, and $\therefore$ DF is parallel to EG; hence $Dy : yF = EC'' : C''G$ $= A'D : B'F$ and the $\angle^s$ $A'Dy$ and $B'FC'$ are equal.

$\therefore \angle A'yD = B'yF$, and as DyF is a right line, $\therefore A'yB'$ must also be a right line; hence the point y must coincide with C', and $\therefore A'C' : C'B' = A'D : B'F = AC : CB$, and $CC' : C'C = CD : DE = AA' : A'A''$, which was to be proved.

The physical question now follows at once, for, let the vertical plane in which the two bodies move intersect the inclined planes in AA'', BB'', and let A and B be the initial positions of the bodies, and A' and B', A'' and B'' any other simultaneous positions; C and C'' being the positions of the centres of gravity corresponding to AB, $A''B''$.

Now since the accelerations down the planes are uniform, $AA' : A'A'' = BB' : B'B''$. Also, by a property of centre of gravity, $AC : CB = A''C'' : C''B''$.

$\therefore$ by preceding theorem, $A'C' : C'B' = AC : CB$, $\therefore C'$ is another position of centre of gravity, which $\therefore$ moves with a uniform acceleration in the right line CC''.

N. B.—The above is only a proof of a particular case of the general question, viz., when the two bodies move in the same vertical plane along the two inclined planes; but the proof of the general question, viz., when the bodies move in any right lines whatever under the action of uniform accelerating forces, starting *simultaneously* from a state of rest, may be completed as follows :—

If AA'', BB'' be *not* in the same plane, then, since $A''E$, $B''G$ are parallel to the same line AB, they are parallel to one another (by Euclid, B. XI. p. 9), and therefore EG and $A''B''$ lie in the plane of $A''E$ and $B''G$ (by Euclid, B. XI. p. 7), and consequently intersect. $\therefore CC''$ lies in the plane ECG, and $\therefore$ intersects DF.

Hence, if any number of bodies start simultaneously from rest along right lines under the action of uniform accelerations, their common centre of gravity will *describe a right* line, and will move with a *uniform acceleration.*

This appears by taking two of the bodies together

then these two and a third body, and so on. The reader who is acquainted with analytic geometry will readily perceive how the question, in its most general state, may be solved by referring the motions to three rectangular axes.

(18.) Let d = diameter, and v = velocity acquired in falling down vertical diameter, then

$$d \cos \theta = \text{chord};$$

$$g \cos \theta = \text{acceleration down chord};$$

$$\therefore v^2 = 2gd$$

$$\left(\frac{v}{n}\right)^2 = 2g \cos \theta \,.\, d \cos \theta$$

$$\therefore 2g \cos \theta \,.\, d \cos \theta = \frac{1}{n^2} \,.\, 2g \,.\, d$$

$$\therefore \cos^2\theta = \frac{1}{n^2}, \text{ and } \cos \theta = \frac{1}{n}. \quad \textit{Ans.}$$

(19.) Let θ = inclination of required radius (r) to vertical diameter, then $g \cos \theta$ = acceleration down radius; hence, substituting in the equation $S = \frac{1}{2}ft^2$, we have

$$2r = \tfrac{1}{2} g \,.\, t^2$$

$$r = \tfrac{1}{2} g \cos \theta \,.\, t^2$$

$$\therefore 2 \cos \theta = 1, \text{ and } \cos \theta = \tfrac{1}{2} = \cos 60^\circ$$

$$\therefore \theta = 60^\circ. \quad \textit{Ans.}$$

(20.) Let θ = inclination of required diameter ($2r$) to vertical diameter, t_1 = time of describing the oblique diameter, t_2 = time of describing its first half, $t = t_1 - t_2$ = time of describing its last half = time of describing vertical diameter,

$$\therefore 2r = \tfrac{1}{2} g \,.\, t^2, \qquad \therefore t = \sqrt{\frac{4r}{g}}$$

$$2r = \tfrac{1}{2} g \cos\theta \,.\, t_1^2, \quad \therefore t_1 = \sqrt{\frac{4r}{g\cos\theta}}$$

$$r = \tfrac{1}{2} g \cos\theta \,.\, t_2^2, \quad \therefore t_2 = \sqrt{\frac{2r}{g\cos\theta}}$$

but $t_1 - t_2 = t$,

$$\therefore \sqrt{\frac{4r}{g\cos\theta}} - \sqrt{\frac{2r}{g\cos\theta}} = \sqrt{\frac{4r}{g}}$$

$$\therefore \frac{4r}{g\cos\theta} - \frac{4r\sqrt{2}}{g\cos\theta} + \frac{2r}{g\cos\theta} = \frac{4r}{g}$$

$$\therefore \cos\theta = \tfrac{3}{2} - \sqrt{2} \left(= \frac{3\sqrt{2}-4}{2\sqrt{2}}\right)$$

$$= 1.5 - 1.4142135 = .0857865$$

$$= \cos 85^\circ\ 4'\ 43''.5. \quad \textit{Ans.}$$

(21.) Let h = height, t = time of descent,

$$\therefore \tfrac{2}{3} h = 16(t-1)^2,$$

$$h = 16t^2,$$

$$\therefore (t-1)^2 = \tfrac{2}{3} t^2,$$

$$\therefore t - 1 = \tfrac{1}{3} t\sqrt{6}, \text{ and } t = \frac{1}{1 - \frac{1}{3}\sqrt{6}},$$

$$= \frac{3}{3 - \sqrt{6}} = 3 + \sqrt{(6)} = 3 + 2.44948$$

$$= 5.44948 \text{ seconds},$$

and $h = 16t^2 = 16(15 + 6\sqrt{6})$

$$= 475.15 \text{ feet.}$$

(22.) Let x = the height above h, which the body attains; then $\frac{1}{2}t$ = time of describing x from rest,

$\therefore$ putting $v = 0$ in equation (5 bis),

we have

$$0 = V^2 - 2g(h + x) = V^2 - 2gH;$$

also $\qquad x = \frac{1}{2} g \left(\frac{1}{2} t\right)^2$ by equation (4),

$$\therefore x = \tfrac{1}{8} gt^2 \text{ and } H = h + x = h + \tfrac{1}{8} gt^2,$$

$$\therefore V^2 = 2gH = 2gh + \frac{g^2 t^2}{4},$$

$$\therefore V = \sqrt{(2gh + \tfrac{1}{4} g^2 t^2)}.$$

(23.) In time t first body falls through h, $\therefore t = \sqrt{\frac{2h}{g}}$;

hence the first body is in motion for $n + \sqrt{\frac{2h}{g}}$ seconds.

$$\therefore S = \tfrac{1}{2} g \left(n + \sqrt{\frac{2h}{g}}\right)^2$$

$$S = Vn + \tfrac{1}{2} gn^2.$$

Equating these two values of S, we have

$$Vn + \tfrac{1}{2} gn^2 = \tfrac{1}{2} g \left(n^2 + 2n \sqrt{\frac{2h}{g}} + \frac{2h}{g}\right)$$

$$\therefore V = \frac{h}{n} + \sqrt{(2gh)}. \quad \textit{Ans.}$$

(24.) The resolved part of Q down the plane is $Q \cdot \frac{h}{l}$;
$\therefore$ the statical force which produces the acceleration f in Q up the plane is

$$f = \frac{P - Q \cdot \frac{h}{l}}{P + Q} \cdot g.$$

Let x = distance required from bottom of plane, v = velocity at this point. Now Q would continue to move

uniformly with velocity v when P is detached, were it not for the retarding force of gravity acting upon it, viz., $g \cdot \frac{h}{l}$; hence by (5 bis) we have for the motion of Q, after P is detached from it,

$$0 = v^2 - 2g\frac{h}{l}(l - x). \qquad (\alpha)$$

Also equation (5) gives for the motion of Q through x,

$$v^2 = 2fx = 2gx \cdot \frac{P - Q \cdot \frac{h}{l}}{P + Q}. \qquad (\beta)$$

From (α) and (β) we have

$$x \cdot \frac{P - Q \cdot \frac{h}{l}}{P + Q} = \frac{h}{l} \cdot (l - x)$$

$$\therefore x = \frac{P + Q}{P} \cdot \frac{hl}{h + l}. \quad \textit{Ans.}$$

(25.) Let x = time after projection of the first body when second must be projected, then by (4 bis) we have

$$S = V_1 t + \tfrac{1}{2} g t^2 \text{ for 1st body,}$$
$$S = V_2(t - x) + \tfrac{1}{2} g (t - x)^2 \text{ for 2nd body;}$$

equating these two values of S, we have

$$\tfrac{1}{2}g(t - x)^2 + V_2(t - x) = V_1 t + \tfrac{1}{2} g t^2$$

$$\therefore t - x = \frac{-V_2 \pm \sqrt{(V_2^2 + 2gV_1 t + g^2 t^2)}}{g}$$

$$\therefore x = \frac{1}{g}\{(gt + V_2) \pm \sqrt{(V_2^2 + 2gV_1 t + g^2 t^2)}\}. \quad \textit{Ans.}$$

Page 116.

(1.) Here, $i = 30^\circ$, $r = 60^\circ$,
$\therefore$ by (31) we have

$$\tan i = e \tan r,$$

or
$$e = \frac{\tan i}{\tan r} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}. \quad \textit{Ans.}$$

(2.) Here, $i = 20^\circ$, $e = .24$

$$\therefore \text{ by (31) } \tan i = e \tan r$$

$$\therefore \tan r = \frac{\tan 20^\circ}{.24}$$

$$\therefore \log \tan r = \log \tan 20^\circ - \log .24$$
$$= 9.5610659 - \bar{1}.3802112$$
$$= 10.1808547 = \log \tan 56^\circ\, 36'$$
$$\therefore r = 56^\circ\, 36'.$$

Also, by (32) we have

$$v^2 = V^2 (\sin^2 i + e^2 \cos^2 i)$$
$$= 10^2\{.3420201^2 + (.24)^2 \times .9396926^2\}$$
$$= 10^2\{.1167 + .0508\}$$
$$= 10^2 \times .1675 \text{ nearly,}$$
$$\therefore v = 10 \times .4093 \text{ nearly,}$$
$$= 4.093 \text{ miles per hour nearly.}$$

(3.) Here $i = 0$, and $\therefore$ by (31) $r = 0$; hence (32) becomes

$$v^2 = V^2 e^2, \text{ or } v = Ve;$$

but since the velocities are uniform, and the spaces the same, the times must be as $v : V$, that is, as $e : 1$, by equation (α).

Page 118.

(1.) Here $M = 10$, $M' = 5$, $e = \frac{1}{3}$, $V = 5$, $V' = 3$, $\therefore$ (35) becomes

$$v = \frac{10 \times 5 + 5 \times 3 - \frac{1}{3} \times 5(5 - 3)}{10 + 5}$$

$= 4\frac{1}{9}$ miles per hour,

$$v' = \frac{10 \times 5 + 5 \times 3 + \frac{1}{3} \times 10(5 - 3)}{10 + 5}$$

$= 4\frac{7}{9}$ miles per hour.

(2.) Here $e = 1$, $\therefore$ (35) becomes

$$v = \frac{10 \times 5 + 5 \times 3 - 5(5 - 3)}{10 + 5}$$

$= 3\frac{2}{3}$ miles per hour;

$$v' = \frac{10 \times 5 + 5 \times 3 + 10(5 - 3)}{10 + 5}$$

$= 5\frac{2}{3}$ miles per hour.

(3.) Here $M = 200$, $M' = 50$, $V = 20$, $V' = 0$, $e = \frac{1}{5}$, $\therefore$ (31) gives

$$v' = \frac{MV + eMV}{M + M'} = \frac{(1 + \frac{1}{5})\, 200 \times 50}{200 + 50}$$

$$= \frac{\frac{6}{5} \times 400}{25} = \frac{6}{5} \times 16 = \frac{96}{5}$$

$= 19.2$ miles per hour.

Page 121.

(1.) In falling through 100 ft. the ball acquires a velocity,

$$V = \sqrt{(2 \times 32 \times 100)} = 80 \text{ ft. by equation } (9),$$
$$\therefore v = eV = 80e \text{ by equation } (30);$$

hence putting $v = 0$ and $V = 80e$ in (5 bis) we have

$$0 = (80e)^2 - 2 \times 34 \times 70$$
$$\therefore e^2 = .7$$
$$\text{and } e = \sqrt{.7} = .836. \quad \textit{Ans.}$$

(2.) If V = velocity acquired in falling through 100 ft. (= 80 ft. by last question), the velocity after fourth hop is

$$v = e^4 V = (\tfrac{1}{2})^4 \times 80 = 5 \text{ ft. by equation } (30);$$

hence putting $v = 0$ and $V = 5$ in (5 bis), we have

$$0 = 25 - 2 \times 32 S \therefore S = \tfrac{25}{64}$$
$$= .390625 \text{ ft.} \quad \textit{Ans.}$$

(3.) In equation (35) make the following substitutions:

$$M = 200, \; M' = 50, \; V = 20, \; V' = -40, \; e = \tfrac{1}{4},$$

$$\therefore v = \frac{200 \times 20 + 50 \times 40 - \frac{1}{4} \times 50\,(20 + 40)}{200 + 50}$$
$$= 5 \text{ miles},$$

$$v' = \frac{200 \times 20 + 50 \times 40 + \frac{1}{4} \times 200\,(20 + 40)}{200 + 50}$$
$$= 20 \text{ miles.} \quad \text{Ans.}$$

(4.) By the question we have

$$v = \mathrm{V} \sin 45^\circ \therefore \text{ by } (32)$$

$$\mathrm{V}^2 \sin^2 45^\circ = \mathrm{V}^2 (\sin^2 i + \tan^2 30^\circ \cos^2 i)$$

$$\therefore \tfrac{1}{2} = \sin^2 i + \tfrac{1}{3} \cos^2 i = \sin^2 i + \tfrac{1}{3} - \tfrac{1}{3} \sin^2 i$$

$$\therefore \sin^2 i = \tfrac{1}{4} \text{ and } \sin i = \tfrac{1}{2}$$

$$\therefore i = 30^\circ;$$

also, by (31),

$$\tan r = \frac{\tan i}{e} = \frac{\tan 30^\circ}{\tan 30^\circ} = 1$$

$$\therefore r = 45^\circ.$$

(5.) Let a, a' represent the velocity of A before and after impact; b, b' of B after the first and second impact; c, c' of C, and d velocity of D after impact.

Hence, putting $e = 1$ in (35), and making the other necessary substitutions, remembering that B, C, D are at rest, at the instant of their first impact, we have

$$\left.\begin{aligned} a' &= \frac{(\mathrm{A} - \mathrm{B})\, a}{\mathrm{A} + \mathrm{B}} \\ b &= \frac{2\mathrm{A}a}{\mathrm{A} + \mathrm{B}} \end{aligned}\right\} \quad (\alpha)$$

$$\left.\begin{aligned} b' &= \frac{(\mathrm{B} - \mathrm{C})\, b}{\mathrm{B} + \mathrm{C}} = \frac{\mathrm{B} - \mathrm{C}}{\mathrm{B} + \mathrm{C}} \cdot \frac{2\mathrm{A}a}{\mathrm{A} + \mathrm{B}} \\ c &= \frac{2\mathrm{B}b}{\mathrm{B} + \mathrm{C}} = \frac{2\mathrm{B}}{\mathrm{B} + \mathrm{C}} \cdot \frac{2\mathrm{A}a}{\mathrm{A} + \mathrm{B}} \end{aligned}\right\} \quad (\beta)$$

$$\left.\begin{aligned} c' &= \frac{(\mathrm{C} - \mathrm{D})\, c}{\mathrm{C} + \mathrm{D}} = \frac{\mathrm{C} - \mathrm{D}}{\mathrm{C} + \mathrm{D}} \cdot \frac{2\mathrm{B}}{\mathrm{B} + \mathrm{C}} \cdot \frac{2\mathrm{A}a}{\mathrm{A} + \mathrm{B}} \\ d &= \frac{2Cc}{C + D} \end{aligned}\right\} \quad (\gamma)$$

Also by the question

$$Aa' = Bb' = Cc' = Dd = \tfrac{1}{4}Aa. \qquad (\delta)$$

Multiplying the values of a', b', and c' in (α), (β), (γ) by A, B, C respectively, striking out the common factor Aa, and equating the results each to $\frac{1}{4}$ by equation (δ), we have the three following equations :

$$\frac{A - B}{A + B} = \tfrac{1}{4} \qquad (1)$$

$$\frac{B - C}{B + C} \cdot \frac{2B}{A + B} = \tfrac{1}{4} \qquad (2)$$

$$\frac{C - D}{C + D} \cdot \frac{2B}{B + C} \cdot \frac{2C}{A + B} = \tfrac{1}{4} \qquad (3)$$

From (1) we obtain

$$B = \tfrac{3}{5} A.$$

Substituting this value of B in (2), we obtain, after some easy reductions,

$$\frac{3A - 5C}{3A + 5C} = \tfrac{1}{3} \therefore C = \tfrac{3}{10} A.$$

Lastly, substituting for B and C in terms of A in (3), we have

$$\frac{3A - 10D}{3A + 10D} = \tfrac{1}{2} \therefore D = \tfrac{1}{10} A;$$

hence the masses of the four bodies are as

$$A : \tfrac{3}{5} A : \tfrac{3}{10} A : \tfrac{1}{10} A,$$

that is, as,

$$10 : 6 : 3 : 1. \quad \textit{Ans.}$$

(6.) In equation (39) put $V' = 0$, $M = M'$, and $e = 1$, then

$$\tan r = \infty \text{ and } \tan r' = 0,$$

$\therefore r = 90^\circ$ and $r' = 0$ or M' moves in the line of impact, and M at right $\angle^s$ to it; and this result being independent of i, shows that the $\angle$ between the paths of two such balls after the shock is always 90°.

(7.) Here $i = 90^\circ - 30^\circ = 60^\circ$, and $i' = 90^\circ + 30^\circ = 120$,

$\therefore$ by (39) we have

$$\tan r = \frac{(M + 2M)\, 2V \sin 60^\circ}{(M - \frac{2}{3} \times 2M)\, 2V \cos 60^\circ + 2M\, (1 + \frac{2}{3})\, V \cos 120^\circ}$$

$$\tan r' = \frac{(M + 2M)\, V \sin 120^\circ}{(2M - \frac{2}{3}M)\, V \cos 120^\circ + M\, (1 + \frac{2}{3})\, 2V \cos 60^\circ}$$

$$\therefore \tan r = -\tfrac{9}{5} \tan 60^\circ, \text{ and}$$

$$\tan r' = \tfrac{9}{5} \tan 60^\circ = \tan 68^\circ\, 56'\, 54'',$$

$$\therefore r + r' = 180^\circ\,;$$

and hence the direction of the motion of each sphere after impact is equally inclined to the common tangent at the point of impact. This inclination

$$= 90^\circ - r' = 90^\circ - 68^\circ\, 56'\, 54'' = 21^\circ.3'.6'.$$

By the first pair of formulæ in (38) we have, since $\sin i = \sin i'$ and $\sin r = \sin r'$,

$$\text{vel. of } M = 2V \cdot \frac{\sin i}{\sin r'} = 2V \times \frac{\sin 60^\circ}{\sin 68^\circ\, 56'\, 54''}$$

$$\text{vel. of } 2M_1 = V \cdot \frac{\sin i}{\sin r'} = 2V$$

$$\therefore v = 2V \times .928$$

$$v' = V \times .928$$

(8.) In (35) putting $e = 1$, and $V' = -V$, we have

$$v = \frac{(M - M')V - M'(V + V)}{M + M'} = \frac{M - 3M'}{M + M'} \cdot V$$

$$v' = \frac{(M - M')V + M(V + V)}{M + M'} = \frac{3M - M'}{M + M'} \cdot V.$$

If M remain at rest, then v must $= 0$, $\therefore M = 3M'$, but if M' remain at rest, v' must $= 0$, and $\therefore M' = 3M$.

Page 127.

(1.) $R = \frac{V^2}{g} \sin 2e = \frac{520^2}{32} \sin 72°$

$= 8450 \times .9510565 = 8036.427425$ ft.

$$T = \frac{2V \sin e}{g} = \frac{520}{16} \sin 36°$$

$= 32.5 \times .5877853 = 19.10302225$ seconds.

(2.) Maximum range $= 2h = \frac{V^2}{g} = \frac{520^2}{32} = 8450$ ft.

And time of flight for this range

$= \frac{1}{4} \sqrt{(8450)} = \frac{1}{4} \times 91.92 = 22.98$ seconds.

(3.) Here $R = \frac{1}{2}$ mile $= 2640$ ft.

$= 2h \sin 2e = 4h \sin e \cos e$,

and greatest height $= 120$ ft. $= h \sin^2 e$,

$$\therefore \frac{h \sin^2 e}{4h \sin e \cos e} = \frac{120}{2640}$$

$$\therefore \tan e = \tfrac{2}{11} \text{ and } \therefore \log \tan e$$
$$= 10 + \log 2 - \log 11 = 9.2596373$$
$$\therefore e = 10^\circ\ 18'.$$

Again, $$R = \frac{V^2}{g} \sin 2e$$

$$\therefore V^2 = \frac{gR}{\sin 2e} = 32 \times 2640 \operatorname{cosec} 20^\circ\ 36'$$
$$\therefore 2 \log V = \log 32 + \log 2640 + \log \operatorname{cosec} 20^\circ\ 36$$
$$- 10 = 5.3804067$$
$$\therefore \log V = 2.6902033,$$
$$\text{and } \therefore V = 490 \text{ ft.}$$

Page 131.

(1.) Here $e = 45^\circ$, $i = 15^\circ$, and $V = 500$ ft.

$$\therefore \text{ by (48), } T = \frac{2V}{g} \cdot \frac{\sin(e - i)}{\cos i}$$

$$= \frac{500}{16} \cdot \frac{\sin 30^\circ}{\cos 15^\circ} = \frac{500}{32} \sec 15^\circ$$

$$\therefore \log T = \log 500 - \log 32 + \log \sec 15^\circ - 10$$
$$= 1.2088762 \ \therefore T = 16.176 \text{ seconds.}$$

Also, $$R = V \,.\, T \frac{\cos e}{\cos i} = 500\, T \cos 45^\circ \sec 15^\circ$$

$$\therefore \log R = \log 500 + 1.2088762$$
$$+ \log \cos 45^\circ + \log \sec 15^\circ - 20 = 3.7723874$$
$$\therefore R = 5920.9 \text{ ft.} = 1.121 \text{ mile.}$$

(2.) $e = 42^\circ$, $i = 12^\circ$, $R = 2\frac{1}{2}$ miles $= 13200$ ft.

$$\therefore \text{ by (52), } R = 4h \frac{\cos e \sin (e+i)}{\cos^2 i}$$

$$= \frac{2V^2}{g} \cdot \frac{\cos e \sin (e+i)}{\cos^2 i}$$

$$\therefore V^2 = \tfrac{1}{2} gR \cos^2 i \sec e \operatorname{cosec} (e+i)$$

$$= 16 \times 13200 \cos^2 12^\circ \sec 42^\circ \operatorname{cosec} 54^\circ$$

$$= 211200 \cos^2 12^\circ \sec 42^\circ \operatorname{cosec} 54^\circ$$

$$\therefore 2 \log V = \log 211200 + 2 \log \cos 12^\circ$$

$$+ \log \sec 42^\circ + \log \operatorname{cosec} 54^\circ - 30$$

$$= 5.5264716 \;\therefore\; \log V = 2.7632358$$

$$\therefore V = 579.74 \text{ ft.} \quad \textit{Ans.}$$

(3.) $i = 10^\circ$, $R = 5280$ ft.
$T = 20$ seconds.

By (52) we have

$$R = \frac{2V^2}{g} \frac{\cos e \sin (e+i)}{\cos^2 i}$$

$$\text{and by (51), } V = \frac{g \cos i \,.\, T}{2 \sin (e+i)}$$

$$\therefore R = \frac{2}{g} \cdot \frac{g^2 \cos^2 i}{4 \sin^2 (e+i)} \cdot \frac{T^2 \cos e \sin (e+i)}{\cos^2 i}$$

$$= \frac{gT^2 \cos e}{2 \sin (e+i)}$$

$$\therefore \frac{\sin (e+i)}{\cos e} = \frac{gT^2}{2R}$$

$$\therefore \tan e \cos i = \frac{gT^2}{2R} - \sin i$$

$$\therefore \tan e \cos 10^\circ = \frac{16 \times 20^2}{5280} - \sin 10^\circ = \frac{40}{33} - \sin 10^\circ$$

$$= 1.038473$$

$$\therefore \log \tan e = \log 1.038473 + \log \sec 10^\circ$$

$$= 10.0230437$$

$$\therefore e = 46^\circ\ 31'. \quad \textit{Ans.}$$

(4.) Here $i = 15^\circ$, $V = 1000$ ft.,
and for maximum range,

$$2e + i = 90^\circ \therefore 2e = 90^\circ - 15^\circ = 75^\circ \text{ and } e = 37^\circ\ 30'$$

$$\therefore \text{by (52)},\ R = \frac{2V^2}{g} \cdot \frac{\cos e \sin (e+i)}{\cos^2 i}$$

$$= \frac{1000^2}{16} \cdot \frac{\cos 37^\circ\ 30' \sin 52^\circ\ 30'}{\cos^2 15^\circ}$$

$$\therefore \log R = 6 - \log 16 + \log \cos 37^\circ\ 30' + \log \sin 52^\circ\ 30'$$

$$+ 2 \log \sec 15^\circ - 40 = 4.6249258$$

$$\therefore R = 42162.4 \text{ ft.} = 7.98 \text{ miles},$$

$$T = \tfrac{1}{4} \sqrt{(\text{maximum range in feet})}$$

$$= \tfrac{1}{4} \sqrt{(42162.4)} = 51.34 \text{ seconds.}$$

(5.) Since the plane is descending, we must change the sign of y in (44),

$$\therefore v^2 = 2g(h + y) = V^2 + 2gy,$$

$$\text{but } y = R \sin i = 42162.4 \sin 15^\circ = 10912 \text{ ft.}$$

$$\therefore v^2 = 1000^2 + 64 \times 10912 = 1698368$$

$$\therefore v = \sqrt{(1698368)} = 1303 \text{ ft.} \quad \text{Ans.}$$

Page 134.

(1.) $V = 1600\sqrt{\frac{3P}{W}} = 1600\sqrt{\frac{27}{196}}$

$= \frac{2400\sqrt{3}}{7} = \frac{2400}{7} \times 1.7325$

$= 594$ ft. *Ans.*

(2.) $V = 1600\sqrt{\frac{3P}{W}} = 1600\sqrt{\frac{6}{48}}$

$= 400\sqrt{2} = 400 \times 1.4142 = 564.68$ ft. *Ans.*

(3.) $V = 1600\sqrt{\frac{3P}{W}} = 1600\sqrt{\frac{18}{32}}$

$= 1600 \times \frac{3}{4} = 1200$ ft. *Ans.*

Pages 134, 135.

(1.) By Example (1) preceding, we have

$$V = 594 \text{ ft.}$$

$\therefore R = \frac{V^2}{g} = \frac{594^2}{32} = 11026$ ft. $= 3675\frac{1}{3}$ yds.;

also, $T = \frac{1}{4}\sqrt{(R \text{ ft.})} = \frac{1}{4}\sqrt{(11026)} = 26\frac{1}{4}$ sec.

(2.) Here P is the quantity sought;

$W = 196$ lbs., $e = 35^\circ$, $i = 10^\circ 40'$, and $R = 4000$ ft.

$\therefore$ by (49), $R = \frac{2V^2}{g} \cdot \frac{\cos e \sin(e-i)}{\cos^2 i}$

$\therefore V^2 = \frac{gR\cos^2 i}{2\cos e \sin(e-i)}$;

substituting this value in (55), we have

$$\frac{gR\cos^2 i}{2\cos e\sin(e-i)} = 1600^2 \times \frac{3P}{W}$$

$$\therefore P = \frac{gRW\cos^2 i}{6\cos e\sin(e-i)\times 1600^2}$$

$$= \frac{32\times 4000\times 196\cos^2 10^\circ 40'}{6\cos 35^\circ\sin 24^\circ 20'\times 1600^2}$$

$$\therefore P = \frac{49}{30}\cos^2 10^\circ 40' \sec 35^\circ \operatorname{cosec} 24^\circ 20'$$

$$\therefore \log P = \log 49 - \log 30 + 2\log\cos 10^\circ 40'$$

$$+ \log\sec 35^\circ + \log\operatorname{cosec} 24^\circ 20' = .6696244$$

$$\therefore P = 4.6733 \text{ lbs.} \quad \textit{Ans.}$$

(3.) $$y = x\tan e - \frac{x^2}{4h\cos^2 e}$$

$$= x\tan e - \frac{x^2}{4h}\sec^2 e = x\tan e - \frac{x^2}{4h}(1+\tan^2 e)$$

$$\therefore x^2\tan^2 e - 4hx\tan e = -x^2 - 4hy$$

$$\therefore \tan e = \frac{2h \pm \sqrt{\{4h(h-y)-x^2\}}}{x}$$

where $x = \frac{3}{4}$ mile $= 3960$ ft.

$y = 250$ ft.

Also, $$V = 1600\sqrt{\frac{3P}{W}} = 1600\sqrt{\frac{9}{32}}$$

$$\textit{and } h = \frac{V^2}{2g} = \frac{V^2}{64} = \frac{9\times 1600^2}{64\times 32} = 11250 \text{ ft.}$$

$$\therefore \tan e = \frac{22500 \pm \sqrt{\{45000(11250 - 250) - 3960^2\}}}{3960}$$

$$= \frac{2250 \pm \sqrt{(450 \times 11000 - 396^2)}}{396}$$

$$= \frac{2250 \pm \sqrt{(4793184)}}{396} = \frac{2250 \pm 2189.33}{396}$$

$$= \frac{4439.33}{396} \text{ or } \frac{60.67}{396}$$

$$\therefore \log \tan e = 11.0496225 \text{ or } 9.1852788$$

$$\therefore e = 84^\circ\, 54' \text{ or } 8^\circ\, 42'. \quad \textit{Ans.}$$

(4.) Here $i = 30^\circ$, $V = 450$ ft., $e = 0$,

$$\therefore \text{by (51), } T = \frac{2V}{g} \cdot \frac{\sin(e + i)}{\cos i} = \frac{450}{16} \tan 30^\circ$$

$$= \frac{75\sqrt{3}}{8} = 16.2 \text{ seconds,}$$

$$\text{and } R = V \cdot T \cdot \frac{\cos e}{\cos i}. \quad \text{(See p. 128.)}$$

$$\therefore R = \frac{450\,T}{\cos 30^\circ} = \frac{450 \times 75\sqrt{3}}{4\sqrt{3}} = \frac{450 \times 75}{4}$$

$$= 8437\tfrac{1}{2} \text{ ft.} = 2812\tfrac{1}{2} \text{ yds.}$$

(5.) The ordinates of the summit of the wall on the edge of the cliff are—

$$p = 500 \text{ ft.} \qquad q = 260 \text{ ft.}$$

and the co-ordinates of the point where the ball falls are—

$$p' = 620 \text{ ft.} \qquad q' = 200 \text{ ft.}$$

$\therefore$ by (53), we have

$$\tan e = \frac{qp'^2 - q'p^2}{pp'(p'-p)} = \frac{4162}{3100}$$

$$\therefore \log \tan e = 10.1279404$$

$$\therefore e = 53^\circ\ 19',$$

and by (54), $V^2 = 2gh = \dfrac{gp^2 \sec^2 e}{2\,(p \tan e - q)}$

$$= \frac{16 \times 500^2 \sec^2 53^\circ\ 19'}{500 \tan 53^\circ\ 19' - 260} = 27251$$

$$\therefore V = 165 \text{ ft.}$$

(6.) By (55) we have

$$V = 1600\sqrt{\frac{3}{32}} = 200\sqrt{6};$$

hence, substituting $2V$ for v in (44), we have

$$-y = \frac{3V^2}{2g} = \frac{18 \times 200^2}{64}$$

$$= 11250 \text{ ft.} = 2.13 \text{ miles.} \quad \textit{Ans.}$$

(7.) Here $R = 50 \text{ yds.} + 6 \text{ ft.} = 156 \text{ ft.}$

$$\therefore R = 4h \sin e \cos e;$$

that is, $39 = h \sin e \cos e.$ (α)

Also, $x = 150$ ft.

$y = 14$ ft.

must satisfy the equation,

$$y = x \tan e - \frac{x^2}{4h \cos^2 e}$$

$$\therefore 14 = 150 \tan e - \frac{150^2}{4h \cos^2 e}$$

$$\therefore 14h \cos^2 e = 150h \sin e \cos e - 5625. \qquad (\beta)$$

By substituting from (α) in (β), we have

$$14h\cos^2 e = 150 \times 39 - 5625 = 225$$

$$\therefore \frac{h^2 \sin^2 e \cos^2 e}{14h\cos^2 e} = \frac{39^2}{225}$$

$$\therefore h\sin^2 e = \frac{14 \times 13^2}{25} = 94.64 \text{ ft.}$$

which by equation (47) is the greatest height.

(8.) Since the range of the projectiles on the hill is the same, we have, by (52),

$$R = 4h_1 \frac{\cos e_1 \sin(e_1 + i)}{\cos^2 i} = 4h_2 \frac{\cos e_2 \sin(e_2 + i)}{\cos^2 i}$$

$$\therefore h_1 \cos e_1 (\sin e_1 + \cos e_1 \tan i)$$
$$= h_2 \cos e_2 (\sin e_2 + \cos e_2 \tan i)$$

$$\therefore \tan i = \tfrac{1}{2} \cdot \frac{h_1 \sin 2e_1 - h_2 \sin 2e_2}{h_2 \cos^2 e_2 - h_1 \cos^2 e_1}. \quad \textit{Ans.}$$